Proceedings of
The 1996 ASME Design Engineering Technical Conferences and
Computers in Engineering Conference
August 18-22, 1996, Irvine, California

96-DETC/DAC-1487

RELIABILITY MODELING IN DESIGN FOR REMANUFACTURE

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ABSTRACT

Remanufacture involves the production-batch disassembly, cleaning, replacement and refurbishment of worn parts in defective or obsolete products. For appropriate products, remanufacture offers significant economic and ecological advantages over other end-of-life options. Since the essential goal of remanufacture is part reuse, the reliability of components is important. The goal of this work is to consider reliability effects on life-cycle costs to enable design for reuse.

A reliability model is developed to describe systems that undergo repairs performed during remanufacture or maintenance. First, the behavior of the model and preliminary experimental verification of the model are described. The model allows replacement of failed parts with both the same and different types of parts. An example simulation applied the reliability model to compare the effects on life-cycle cost of various combinations of mechanical components in a series system.

MOTIVATION

In addition to resource conservation, design for product end-of-life is compelled by existing and impending product take-back laws that place product end-of-life responsibility on the manufacturer. Given this responsibility, the manufacturer may choose to pay increasingly higher fees for landfill or incineration, or have the product reused, repaired, remanufactured, or recycled for scrap material. While many design-for-end-of-life guidelines emphasize facilitating scrap-material recycling, significant resources are consumed during the recycling process. Furthermore, material degradation often results due to molecular breakdown and contamination, both of which are frequently characteristic of current recycling technologies. Individual product repair and maintenance are limited by the high labor costs that tend to cause discarding of repairable products. Lund (1983) observed that by recycling at the component instead of the material level, remanufacturing avoids the possibly unnecessary resource consumption of scrap-material recycling while preserving the value added to the component during manufacture. Also, the production-batch and off-site nature of remanufacturing results in a labor cost significantly lower than that required for individual repair.

While remanufacture is not suitable for all products, it is particularly appropriate for technologically stable items, where a large fraction of components can be reused after refurbishment. Product design that facilitates any of the steps of remanufacture, namely disassembly, sorting, cleaning, refurbishment, reassembly and testing, will facilitate remanufacture. However, the essential goal in remanufacture is part reuse. If a part cannot be reused as is or after refurbishment, the ease of disassembly, cleaning or reassembly will not matter. Refurbishment activities aim to return a part to a like-new or better condition, and include, for example, reboring out-of-round cylinders or fitting cylinders with sleeves.

When parts are to be reused, in either remanufacture or maintenance, the reliability of the part is very important. Collaboration was initiated with three companies that remanufacture a variety of products to learn about the remanufacture process and how products can be designed to facilitate remanufacture. These companies are Eastman Kodak, a manufacturer and remanufacturer of photocopiers, single-use cameras, and medical analysis equipment; Nashua Cartridge Products, a remanufacturer of toner cartridges; and Arrow Automotive Industries, a remanufacturer of automotive after-market parts. This collaboration offered insights on reliability issues across the companies and needs for reliability modeling.

Existing reliability models are unsuitable for describing systems that undergo repairs performed during remanufacture. The goal of this research is to develop and verify reliability models to be used in life-cycle cost estimations of systems where reuse of working components is possible. These calculations help explore initial part design and remanufacture process plan alternatives in the context of other life-cycle concerns. Currently, this model is used in a genetic-algorithm based optimization of life-cycle costs. This paper focuses on the reliability modeling and simulation.

OVERVIEW

This paper begins by highlighting related work in the fields of both life-cycle design and reliability. Selected reliability models with features closest to those desired are detailed before introducing the motivation for distinct characteristics incorporated into the model developed here. The goal of this paper is to illustrate properties of this model and how it can be applied to compare design alternatives for mechanical series systems. This model currently describes series systems whose components have Weibull-distributed densities of time to failure. Therefore Weibull-related terminology and notation are first defined. The model simulates the replacement of failed parts with components of the same or a different type. Replacement parts can be either new or remanufactured. Parts of the same type are those that have identical failure characteristics to the original part. The simulation results of replacement with the same parts were experimentally verified. Replacement of failed parts by components of a different type often more accurately portrays remanufacture, where the replacement part has different failure properties from those of the original part. The modification can be subtle, due to different sources for replacement parts, or drastic, due to reconfiguration of the system for upgrades or correction of known reliability problems. The interaction of multiple parts in a system is described using series-system reliability theory. An example applies the model to compare the life-cycle costs of various combinations of mechanical elements and illustrates additional considerations for application to mechanical systems.

LITERATURE REVIEW

Researchers have considered the roles of failure and serviceability in life-cycle design. Gershenson and Ishii (1991) implemented Service Mode Analysis, which focuses on repair possibilities of various system malfunctions, in a computer tool that calculates serviceability indices of various user-defined failure phenomena. DiMarco et al. (1995) integrated Failure Modes and Effects Analysis into a computer-aided design tool to bring consideration of service costs early in the design process.

Ascher and Feingold (1984) surveyed the considerable body of reliability literature on repairable systems and observed that much of this work models one of two extremes. The first extreme represents the repair process as returning a system to a same-as-new condition. The other extreme, known as minimal repair or same-as-old, describes the system reliability after repair as identical to that before failure. This is rationalized by repairs that involve the replacement of a small fraction of a system's parts. For this model to be exact, the replacement part must have the same distribution of time to failure as the original part, and the same age if the failure rate is age dependent.

Moderation of the above two extremes include the following. Brown and Proschan (1980) model imperfect repair, where at each repair, renewal to the same-as-new state occurs with probability P and no age reduction or same-as-old occurs with probability $1-P$. Nakagawa (1980) models partial renewal of a system at maintenance times, where the effective unit age is reduced to a proportion of the actual age. De la Mare (1979) fit Weibull distributions to data for successive times between failures for many types of systems. He used the estimated means in a cost model to optimize system life-cycle costs. Cozzolino (1968) developed two models, the n-component device model and the time accumulation model. The n-component device model tracks the ages of a system's constituent parts, and each part can have different distributions of time to failure. The time accumulation model, developed to reduce the complexity of the n-component device model, assumed that the n constituent parts are identical.

Cozzolino's n-component device and time accumulation models have some desirable properties which will underlie the unique features of the model developed here. The Cozzolino models assume neither complete system renewal to the same-as-new condition nor minimal repair to the same-as-before-failure state, a trait retained here.

The n-component device model describes a system composed of n parts in series, such that failure of any one part results in system failure. Each part's failure characteristics are independent of other parts' failure processes. Time to first system failure is the minimum of the components' times to first failure. The device ages by accumulating time on its constituent parts, and the vector of component ages determines the density of future time to failure. Failure of one part is repaired by replacement with a part of the same type. Since only the age of the replaced part is reset to zero while the other components retain their age, the system failure rate never returns to its initial value. The time accumulation model produces behavior similar to the n-component device model in a less structured manner by assuming n identical parts, so that the identity of the repaired part need not be tracked. At each failure, $1/n^{th}$ of the system accumulated age is lost.

CHARACTERISTICS OF MODEL

The model developed here describes a population of n-component series systems. The n parts have independent and different distributions of time to failure. The population of n-component systems is represented as a collection of n populations of constituent components, and parts are treated as members of their respective populations. Populations of "single-component" systems are used to introduce this model.

The age distributions of each of the part populations are tracked to determine the reliability of the composite system population. Time-to-failure and age distributions associated with each part population are used to calculate the probability of failure of that part at any given time. Failure of a part has different consequences. First, the failed part can be replaced with a part of the same type while retaining the working parts of the system. Second, the failed part can be replaced by a part of a different type, while the rest of the system either remains unchanged or is reconfigured to accommodate the replacement part. Finally, a failed part can cause replacement of the entire system with either an identical or a different system.

The possibility of system modification is not included in many models, including those of Cozzolino, in which replacement is limited to a component of the same type. This additional capability is motivated by common practices in remanufacture. For example, bearings are often replaced with higher-durability bearings during remanufacture. Many refurbishment processes change the reliability characteristics by altering the system configuration. For example, bronze bushings are installed in distributor housings that wore due to the lack of separate bearings in the original design.

In this model, the repair policy determines actions executed upon part failure. In practice, corporate refurbishment policy significantly affects both the system reliability and the consequent remanufacture cost of a given original design. Some companies may choose to always replace a particular part without inspection, either due to product reconfiguration or past reliability problems, while others will replace based on either actual part failure or projected remaining life.

This model describes series systems where the density of time to failure of each component is represented by the two-parameter Weibull distribution. The extension of this model to use other distributions is fairly straightforward. The Weibull distribution was selected because it is appropriate for many engineering applications. Special cases of the Weibull distribution include the exponential and Rayleigh distributions.

WEIBULL DISTRIBUTION NOTATION

The Weibull probability density function is:

$$f(x) = \frac{\beta x^{\beta-1}}{\theta^{\beta}} \exp[-(\frac{x}{\theta})^{\beta}], \quad 0 \le x \le \infty \tag{1}$$

where x is a random variable that can represent time.

Fig. 1 plots Weibull distributions with θ=10 and values of β from 1 to 5. β is the shape parameter; a larger β reduces spread about the expected value. θ is the scale parameter; as β increases, the peak of the distribution approaches θ. A part with a Weibull-distributed density of time to failure has an expected lifetime of approximately θ. β indicates the certainty of the expected value. β=1 yields the exponential distribution, and β=2, the Rayleigh distribution. The effects of β and θ on the model output will appear throughout the paper.

The value of f(x) is the probability that a part fails between times x and x+dx. F(x), the integral of f(x) from 0 to x, represents the probability that a part fails at any time up to x. 1-F(x) is the probability that the part will survive past x.

$$F(x) = \int_0^x f(x)dx \tag{2}$$

Fig. 2 plots F(x) curves corresponding to the densities of Fig. 1. Note that θ locates the intersection of the family of integrals.

Another quantity often used in reliability is the failure rate function which is represented by:

$$\lambda(x) = \frac{f(x)}{1 - F(x)} \tag{3}$$

The failure rate is the conditional probability of failure at x given survival to x. Note that the failure rate becomes greater than 1, and that the failure rate function is not a density function. Fig. 3 plots failure rates corresponding to the distributions of Fig. 1. The negative exponential distribution (β=1) yields a constant failure rate. The failure rate for Weibull densities increases with x to the power of (β-1).

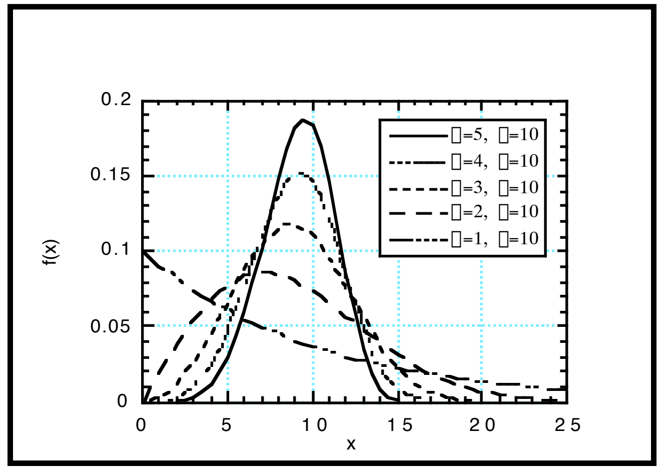

Fig. 1. Weibull probability distributions

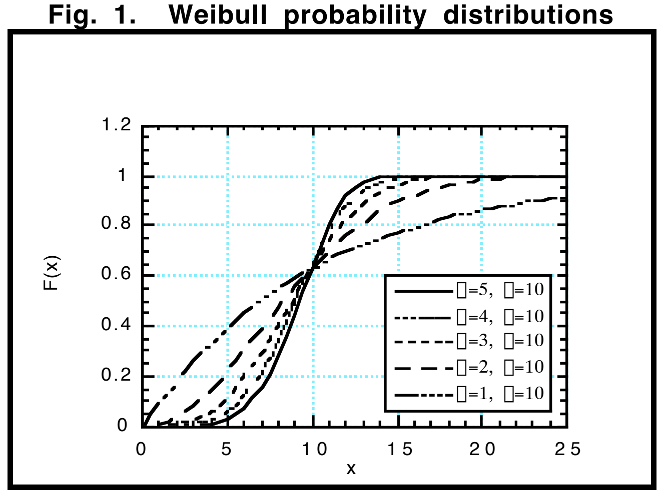

Fig. 2. Weibull probability of failure

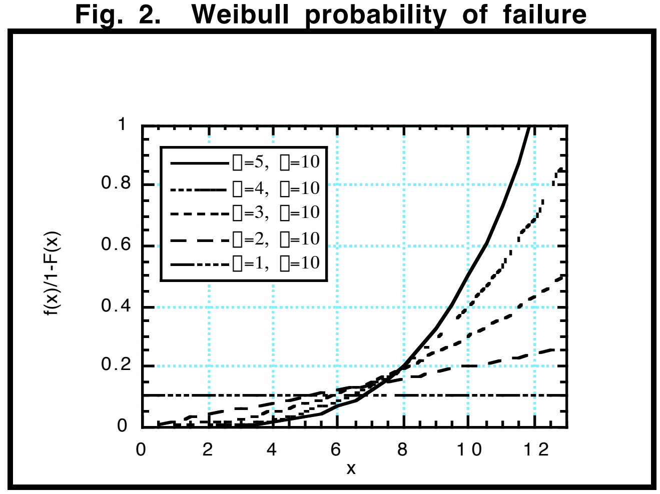

Fig. 3. Weibull failure rate

SIMULATION

The density function of time to failure of each part of a system is used to calculate the life-cycle cost for a population of systems. An age distribution is obtained at each time step for each part population. The age distribution determines failure rates for the following time step. The failure rates of each part determine the replacement part cost portion of the life-cycle cost of the system. Failed parts can be replaced with parts of either the same or different type. First presented will be the simulation of replacing failed parts by the same type of parts. The model behavior for this basic simulation is experimentally verified. Next presented is the simulation of replacing failed parts with components of a different type. This section examines the behavior of the model for a single part population. The following section will describe how the interactions between multiple parts of a system are treated.

Description of Basic Simulation

Age bins are used to track the age distribution of a population of parts. The time-to-failure density determines the portion of the contents of each bin that survive to the next time step, appearing as contents for the next older bin, and the portion that fails, appearing as contents in the zero-age bin. Figs. 5a through 5f track the age bin distributions for six consecutive time steps for a population of parts whose time-to-failure density and corresponding probability of failure are plotted in Fig. 4. Age bins are created at increments equal to the time between events, e.g., number of years between remanufacture activities. Time between events, or bin size, of 1 was used to produce results shown in Figs. 5a through 5f.

Initially, all parts are in the first bin as shown in Fig. 5a: the population consists only of new parts. That is, at $t_0 = 0$, $q_0(t_0) = 1$, where q_i is the fraction of parts in the ith bin.

At the next time step, the failure density is integrated using numerical methods developed by Senin et al. (1996) from zero to one time increment to find the probability of failure. The portion of the population that survives advances to the next age bin, and the portion that fails is replaced and reappears as items in the first age bin, as shown in Fig. 5b. That is, at $t_1 = \Delta t$, the fractions of the first two bins become:

$$\begin{aligned} q_1(t_1) &= q_0(t_0)[1 - \int_0^{\Delta t} f(x)dx] \\ q_0(t_1) &= q_0(t_0)[\int_0^{\Delta t} f(x)dx] \end{aligned} \qquad (4)$$

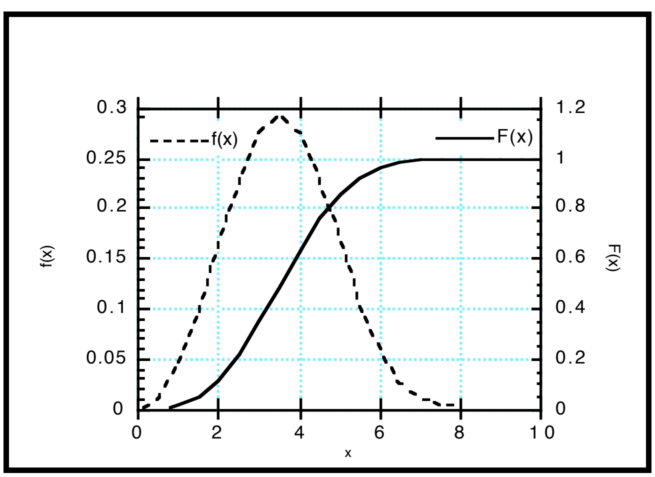

Fig. 4. Time to failure and probability of failure for Weibull parameters □=3, □=4.

Again, portions of both age bins survive and advance to the next age bin, and portions of failed parts from both bins appear as replaced parts in the first bin. The proportions of each bin for $t_2 = 2\Delta t$, the following time step, are calculated by:

$$q_2(t_2) = q_0(t_0)[1 - \int_0^{2\Delta t} f(x)dx]$$

$$q_1(t_2) = q_0(t_1)[1 - \int_0^{\Delta t} f(x)dx] \qquad (5)$$

$$q_0(t_2) = q_0(t_0)[\int_{\Delta t}^{2\Delta t} f(x)dx] + q_0(t_1)[\int_0^{\Delta t} f(x)dx]$$

Finally, at $t_n = n\Delta t$, the fractions of parts in each bin are:

$$q_n(t_n) = q_0(t_0)[1 - \int_0^{n\Delta t} f(x)dx]$$

$$q_{n-1}(t_n) = q_0(t_1)[1 - \int_0^{(n-1)\Delta t} f(x)dx]$$

$$\ldots$$

$$q_2(t_n) = q_0(t_{n-2})[1 - \int_0^{2\Delta t} f(x)dx]$$

$$q_1(t_n) = q_0(t_{n-1})[1 - \int_0^{\Delta t} f(x)dx] \qquad (6)$$

$$q_0(t_n) = q_0(t_0)[\int_{(n-1)\Delta t}^{n\Delta t} f(x)dx] + q_0(t_1)[\int_{(n-2)\Delta t}^{(n-1)\Delta t} f(x)dx] + \ldots + q_0(t_{n-2})[\int_{\Delta t}^{2\Delta t} f(x)dx] + q_0(t_{n-1})[\int_0^{\Delta t} f(x)dx]$$

For the preceding simulation, much of the initial population advances to the next age bin for the first two time steps. At time t_3, over 60 percent of the parts put into service at time t_o are still in service. By time t_4, under 40 percent of the original parts have survived, and by time t_5, fewer than 15 percent of the original parts are still in service. This can be inferred from the probability of failure, F(x) of Fig. 4; F(x) is initially very small, but by t_5, it is about 85 percent.

The average age of the population is calculated by summing over all the age bins the product of the fraction of parts in that bin and the age of the bin:

$$\bar{a}(t) = \sum_{i=0}^{n(t)} q_i(t) a_i \qquad (7)$$

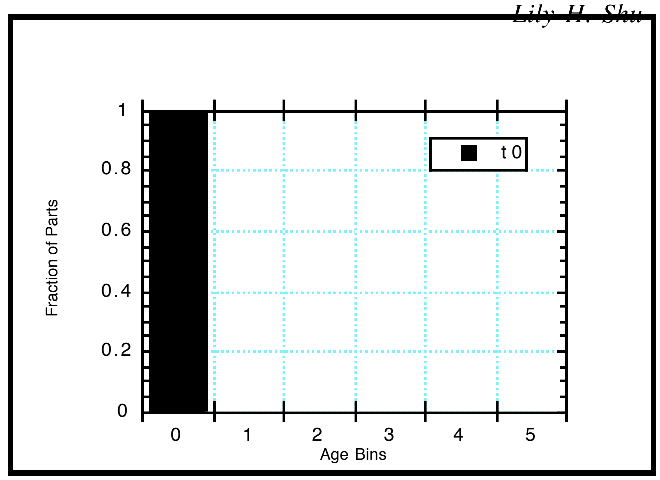

Fig. 5a. Bin Distribution at $t_o = 0$

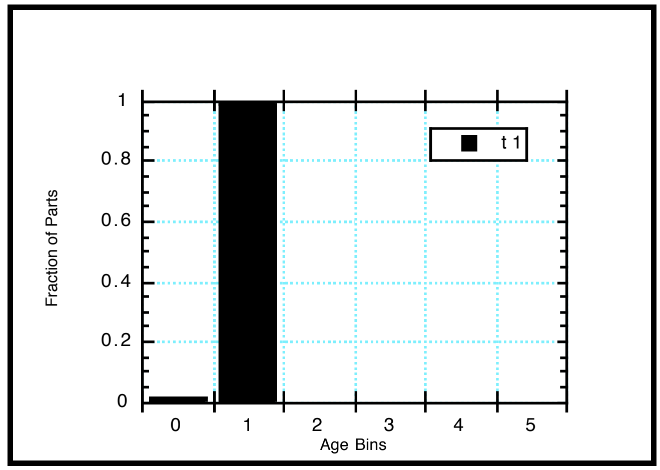

Fig. 5b. Bin Distribution at $t_1 = \Delta t$

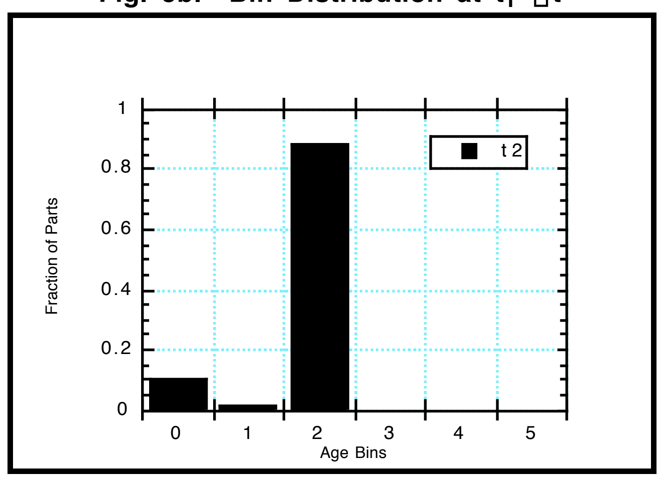

Fig. 5c. Bin Distribution at $t_2 = 2\Delta t$

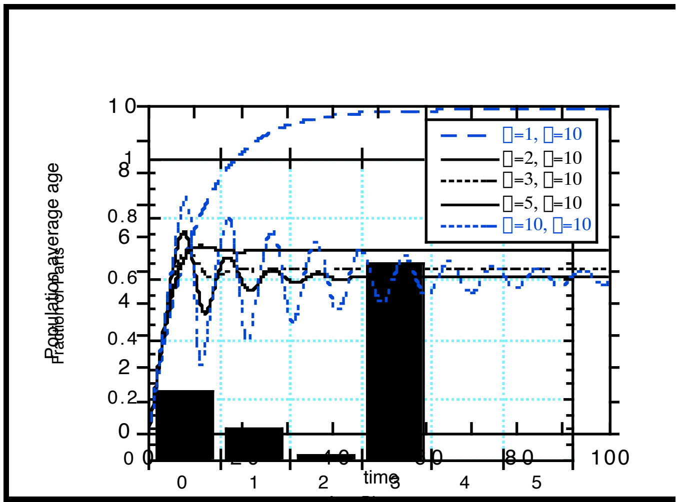

Fig. 6a. Population average age

Fig. 5d. Bin Distribution at t_3=3□t

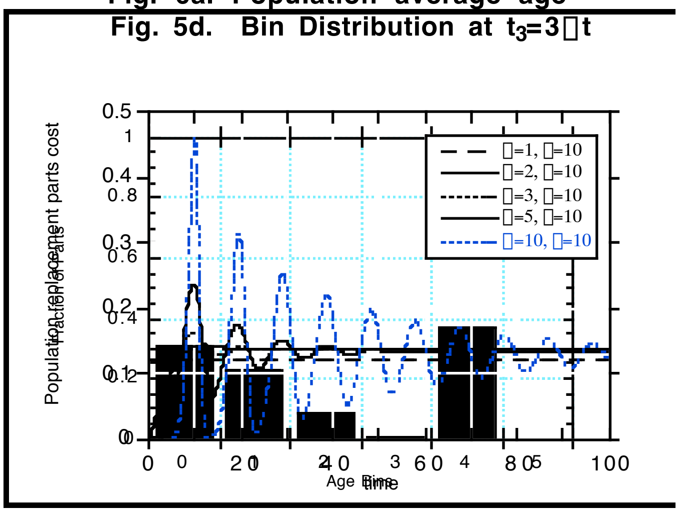

Fig. 6b. Population replacement parts cost

Fig. 5e. Bin Distribution at t_4=4□t

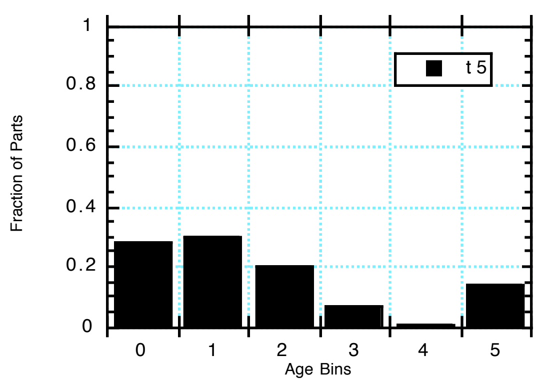

Fig. 5f. Bin distribution at t_5=5□t

Results of Basic Simulation

Fig. 6a plots the average age of constant-size populations of identical parts that are replaced by components of the same type upon failure. Each curve represents a population of parts with a particular Weibull distribution of time to failure. The plots shown correspond to a constant value of □=10 paired with □=1,2,3,5, and 10. Both the horizontal and vertical axes have the same units of time, e.g., minutes, hours, or years.

Several characteristics of Fig. 6a are of interest. First, the average age eventually reaches a steady state value. This is in agreement with Drenick's Theorem (Drenick 1960), which states that the superposition of an infinite number of independent equilibrium renewal processes is a homogeneous Poisson process. A homogeneous Poisson process is one that can be represented by an exponential distribution. Recall from Fig. 3 that the failure rate corresponding to the exponential distribution (□=1) is constant. A population with a constant failure rate and part renewal upon failure has a constant average age. The value of the steady state age depends upon Weibull parameters □ and □. The dependence on □ is not surprising; higher values of □ for a given set of □'s yield higher values for expected time to failure and thus average age.

Alpha affects both the steady state value and the degree of oscillation. Recall from Figs. 1 through 3 that as □ increases, the window of time during which a majority of parts fail decreases. For high values of □, very few parts will fail until time=□, at which time almost all the parts will fail immediately. During the low-failure period, the average age will increase monotonically. Then as increasingly large numbers of parts fail, the replacement of a significant portion of the population causes the average age to drop until the wave of failure is over. The newly installed base of parts then ages steadily until the next failure wave. During each oscillation, a number of parts fail outside the time window during which most of the population fail. The population thus becomes more age-diversified with each cycle, and the oscillations in average age die down. The higher the value of □, the fewer parts fail outside the tighter expected failure period, and thus the greater the oscillations in average age and the longer it takes for age diversification to occur. As □ increases, the mean of the average age approaches □/2. This is intuitive when one considers the upper bound as □ approaches infinity. Physically, such a distribution of time to failure implies that no parts fail until time=□, at which time all the parts fail. Therefore this population would have a saw-toothed average age plot that does not decay and is bounded between 0 and □.

Fig. 6b plots the replacement parts cost corresponding to Fig. 6a. For ease of comparison, all parts were assigned identical costs. In reality, cost is likely to be a function of both □ and □. The trends of Fig. 6b are consistent with those of Fig. 6a. The total replacement part cost increases as average age decreases since parts are being replaced at a higher rate. Steady state replacement costs are higher for lower steady state average ages.

Experimental Verification of Basic Simulation

An experiment was performed to verify the basic behavior of the model. This experiment applied the model to a fastening system and involved obtaining data on the number of

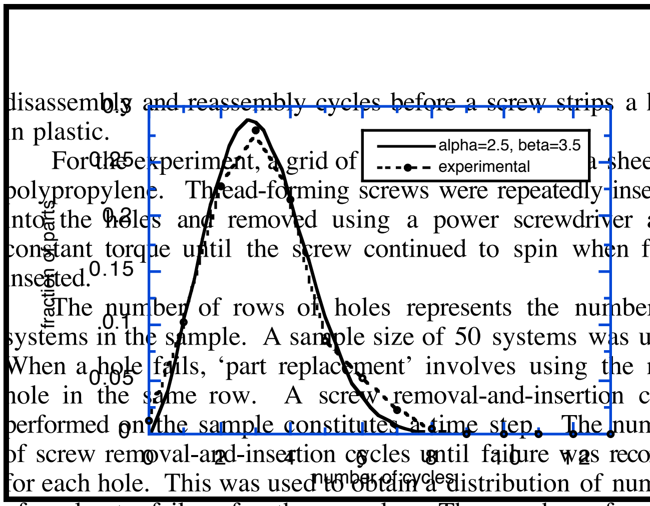

Fig. 7a. Sample cycles-to-failure histogram vs. Weibull distribution with α=2.5, β=3.5

disassembly and reassembly cycles before a screw strips a hole in plastic.

For the experiment, a grid of a sheet of polypropylene. Thread-forming screws were repeatedly inserted into the holes and removed using a power screwdriver at a constant torque until the screw continued to spin when fully inserted.

The number of rows of holes represents the number of systems in the sample. A sample size of 50 systems was used. When a hole fails, 'part replacement' involves using the next hole in the same row. A screw removal-and-insertion cycle performed on the sample constitutes a time step. The number of screw removal-and-insertion cycles until failure was recorded for each hole. This was used to obtain a distribution of number of cycles to failure for the sample. The number of cycles survived by each active hole averaged over the sample at each time step yields the average age plot. The data associated with the final holes were not used to obtain the cycles-to-failure distribution because those holes had not failed yet, but they were used to calculate the average age.

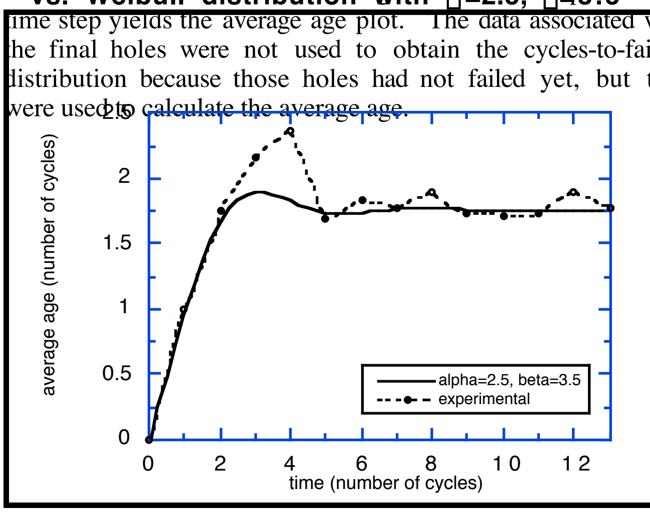

Fig. 7b. Average age obtained experimentally vs. through simulation using Weibull distribution with α=2.5, β=3.5

Fig. 7a compares the sample histogram of cycles to hole failure with the Weibull distribution that produced the least-squared error between the experimental data points and the values determined by the distribution. Fig. 7b compares the average age yielded experimentally with that produced through simulation using the above Weibull distribution.

In a second experiment, a higher torque was used to obtain a different distribution of cycles to failure of the holes. Higher torque reduces □ by stripping the hole in fewer cycles. A less controlled and predictable process also reduces □. Fig. 8a compares the sample histogram of cycles to hole failure with the Weibull distribution producing the least squared error. Fig. 8b compares the experimental average age with the average age simulated using the corresponding Weibull distribution.

For both experiments, the agreement is reasonable considering the relatively small sample size of 50. The smaller sample size is much more sensitive to outliers and thus displays a greater noise level than if a larger sample were used.

Simulation of System Modification

The preceding simulation results and experimental verification were for the replacement of failed parts with parts of the same type. As explained earlier, repairs during remanufacture often change the reliability characteristics of a system by replacing failed parts with components of a different type. The remaining parts of the system can either stay the same or be reconfigured to accommodate the replacement part. Similar experimental verification of this behavior could involve using, in the first hole of each row, screws with thread densities different from the screws used in the remaining holes of each row. The different thread density will result in a different distribution of disassembly and reassembly cycles to failure for identical holes. In the remanufacture of toner cartridges, when a plastic boss is stripped, a larger or coarser-thread screw is often used in place of the original screw.

The simulation results for the replacement of failed parts of a population with parts of a different type follow. Subsequent failure of replacement parts result in replacement by the same parts, i.e., parts of the original type are not reintroduced into the population.

Figs. 9a and 9b chart the replacement of an initial population of parts with Weibull parameters α=3, β=10, denoted (3,10), with parts of Weibull parameters α=10, β=10, denoted (10,10). Subsequent replacement of failed (10,10) parts are with the same (10,10) parts. For reference, replacement of an initial population of (3,10) parts by the same (3,10) parts and replacement of an initial population of (10,10) parts by the same (10,10) parts are also plotted. Of interest in the average age and part replacement cost plots are the phase shift and reduced oscillation of the (3,10)-to-(10,10) curve relative to the (10,10)-to-(10,10) curve. An original population of (3,10) parts fail earlier and with more spread between time-of-failures than an original population of (10,10) parts. Therefore the first replacement batch of (10,10) parts appear earlier and more staggered over time for a population that began with (3,10) parts than for a population that began with (10,10) parts. The effect of this initial difference carries over to subsequent replacement cycles.

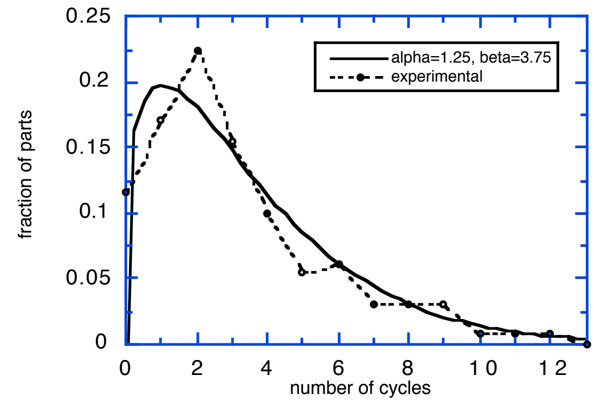

Fig. 8a. Sample cycles-to-failure histogram vs. Weibull distribution with α=1.25, β=3.75

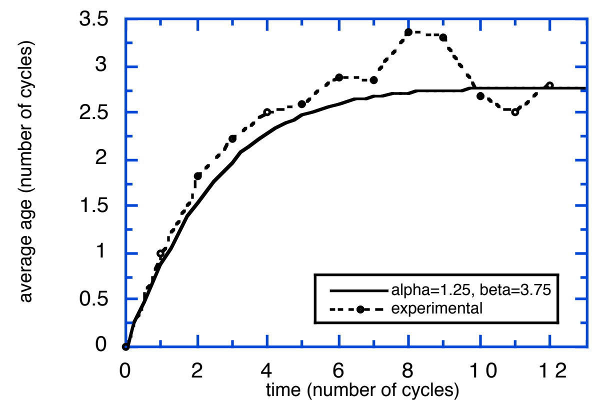

Fig. 8b. Average age obtained experimentally vs. through simulation using Weibull distribution with α=1.25, β=3.75

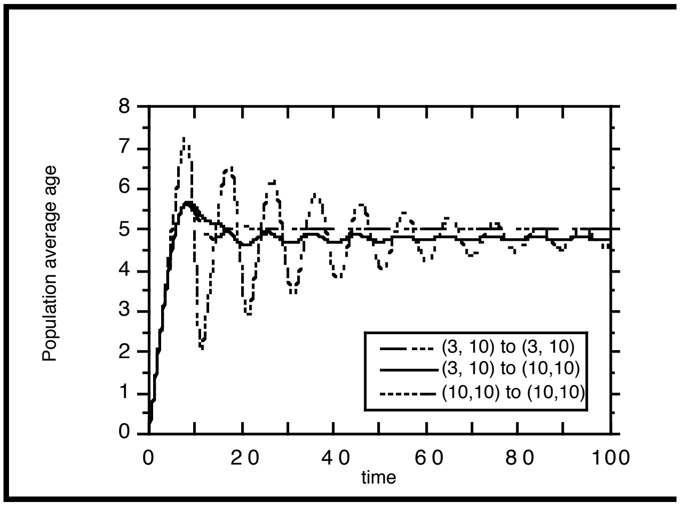

Fig. 9a. Population average age

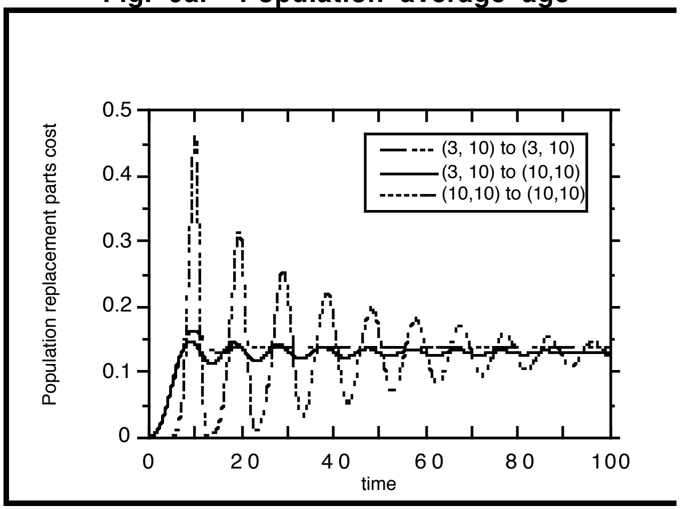

Fig. 9b. Population replacement parts cost

SERIES SYSTEM BEHAVIOR

The previous sections described the behavior of the model for single populations of parts. This section will illustrate how the reliability of a system is obtained from the reliability of the constituent parts in series. In a series system, the failure of any one of the constituent parts results in system failure.

The failure rate of a series system is the sum of the failure rates of the components:

$$\lambda_{sys}(x) = \sum_{i=1}^{N} \lambda_i(x) \tag{8}$$

The reliability of a series system is the product of the reliability of the components:

$$R_{sys}(x) = 1 - F_{sys}(x) = \prod_{i=1}^{N} 1 - F_i(x) \tag{9}$$

From (3), the failure density of a series system with parts with Weibull failure densities is then:

$$f_{sys}(x) = \lambda_{sys}(x) R_{sys}(x) = \sum_{i=1}^{N} \frac{\beta_i x^{\beta_i - 1}}{\theta_i^{\beta_i}} \prod_{i=1}^{N} e^{-(\frac{x}{\theta_i})^{\beta_i}}$$

$$f_{sys}(x) = \sum_{i=1}^{N} \frac{\beta_i x^{\beta_i - 1}}{\theta_i^{\beta_i}} (e^{-\sum_{i=1}^{N} (\frac{x}{\theta_i})^{\beta_i}}) \tag{10}$$

For example, consider a system composed of two parts in series, each with a density of time to failure that is described by a Weibull distribution with parameters, β=3, θ=10:

$$f_1(x) = f_2(x) = \frac{3x^2}{10^3} \exp\left[-(\frac{x}{10})^3\right] \tag{11}$$

The density of time to failure for this system is:

$$f_{sys}(x) = 2(\frac{3x^2}{10^3}) \exp\left[-2(\frac{x}{10})^3\right] \tag{12}$$

The probability of system failure can be obtained by integrating (12). For a two-component system, the probability of failure can also be computed by:

$$F_{sys}(x) = F_1(x) + F_2(x) - F_1(x)F_2(x) \tag{13}$$

where the probabilities of part failure are obtained by integration of the corresponding part failure density functions.

System failure can result in either partial or complete replacement of the system. Figs. 10a and 10b compare the average age and replacement part cost for replacement of only the failed part versus system replacement. As expected, the average system age is higher if only the failed part is replaced, and the replacement cost is lower if only the failed part is replaced. Components of a system are sometimes arranged or joined in a manner that requires the replacement of more than one part upon the failure of a single part. Also, part consolidation often results in single parts containing multiple features, the failure of any one of which would require part replacement. The cost curves of Fig. 10b suggest the advantages of making failure-prone features separable, so that the failure of a small portion of a part does not require the replacement of a largely unaffected and possibly expensive part.

SERIES MECHANICAL SYSTEMS

This section presents additional considerations for application of the model to mechanical systems. The model is then applied to an example mechanical system to compare life-cycle part replacement costs for various combinations of component selection.

Wear and failure of mechanical components often occur due to relative motion between parts, and thus the reliability of many mechanical components depends on the interaction with the component with which it is coupled. For example, a gear may have different failure characteristics depending on the gear with which it meshes. Therefore, failure characteristics are defined as interactions between parts.

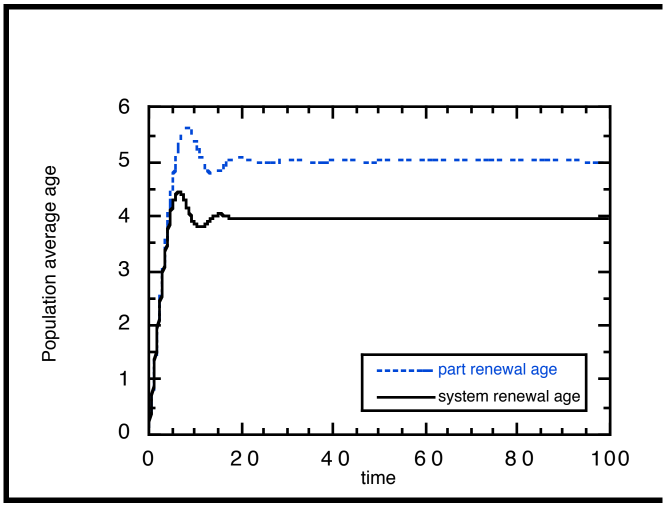

Fig. 10a. Average age: part vs. system renewal

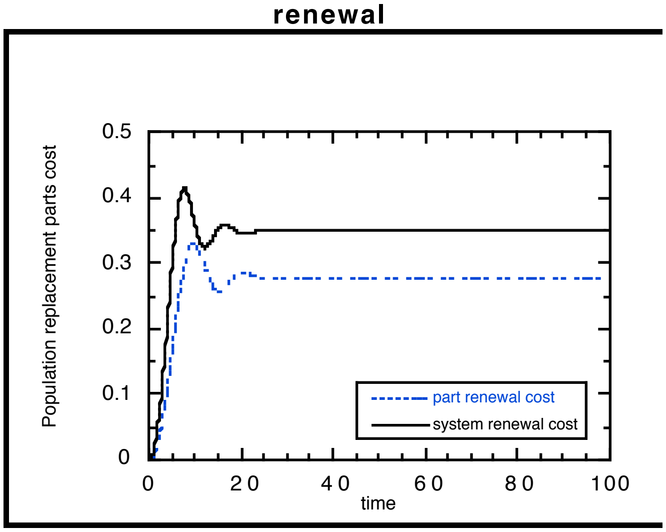

Fig. 10b. Part cost: part vs. system renewal

For example, consider the driver-shaft-and-bevels assembly and driven bevel pinions illustrated in Fig. 11. The interactions between the driver and driven bevels, as well as the material and geometry characteristics of each gear, prescribe the gear failure parameters. Tables 2 and 3 contain hypothetical gear failure characteristics as □ and □ of the Weibull distributed time-to-failure density. Table 2 contains failure density functions of driver-assembly bevels made from three different materials as a function of the material of the meshing driven bevel pinion. Table 3 contains the corresponding failure characteristics for the driven bevels. The trend assumed by the tables is that a softer material wears faster when meshed with a harder material.

If the entire driver assembly is replaced as a unit upon failure of either driver bevel, the assembly has two simultaneous interactions. The resultant failure density of the assembly due to the failure of either driver bevel can be found using (10).

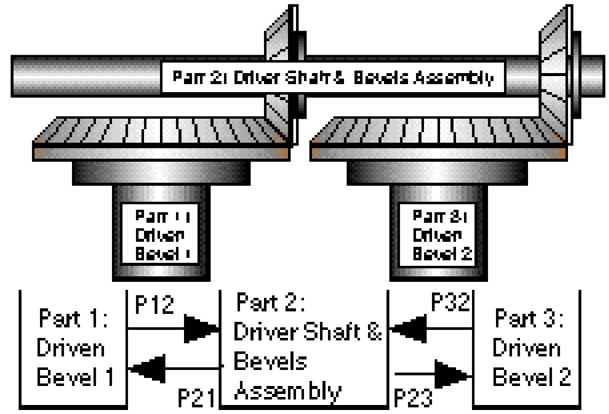

Fig. 11. Failure characteristic representation for mechanical elements in series

Table 1. Gear costs used in simulation

Gear material	Driven Bevel 1	Driver shaft/bevel Assembly	Driven Bevel 2
Polished steel	20	50	20
Brass	15	40	15
Nylon	5	15	5

Table 2. Failure distributions of driver assembly bevels for various material combinations

Driver bevel material	*Driven bevel material*		
	Polished steel	*Brass*	*Nylon*
Polished steel	□=6, □=8	□=7, □=12	□=8, □=16
Brass	□=3, □=4	□=4, □=8	□=5, □=12
Nylon	□=1, □=1	□=2, □=2	□=2, □=3

Table 3. Failure distributions of driven bevels for various material combinations

Driven bevel material	*Driver bevel material*		
	Polished Steel	*Brass*	*Nylon*
Polished steel	□=6, □=16	□=7, □=24	□=8, □=32
Brass	□=3, □=8	□=4, □=16	□=5, □=24
Nylon	□=1, □=2	□=2, □= 4	□=2, □=6

The part cost and failure data of Tables 1 through 3 are used to compare life-cycle part replacement costs for four combinations of part selection. These combinations are: steel driver-assembly bevels with steel driven bevels, steel driver-assembly bevels with brass driven bevels, brass driver bevels with nylon driven bevels, and nylon driver bevels with nylon driven bevels. In each combination, both the driver bevels are of the same material, as are both the driven bevels.

Several simplifications over typical practice are made. The effects of the attachment between the bevels and the shafts are neglected. Meshing gears are usually both replaced when either needs to be replaced, but here only the failed part is replaced, and the failure characteristics of one gear are assumed to be independent of the meshing gear age. The driver-shaft-and-bevels assembly is counted as one part and replaced as a unit.

The cumulative part costs for the above material combinations, shown in Fig. 12, suggest that the use of cheaper parts is more cost effective. However, the cumulative cost included only part costs, not labor cost, nor the cost of disruption while the failed part is being replaced. Fig. 13 plots the total population replacement part costs obtained by adding a uniform cost of 60 to the part costs in Table 1. This additional cost can either represent a labor cost incurred each time a part is installed or replaced, or make up for an initial underestimate of component costs. The results are then reversed: the most cost effective combinations are those that incur a larger part cost, but also last longer. This confirms that the labor and disruption costs associated with replacing a component, in addition to the cost of the component, should drive initial component selection. Figs. 12 and 13 also suggest that the interaction between part cost and reliability makes it difficult to predict from intuition alone the combination of component selection that will yield the lowest life-cycle cost.

SUMMARY AND FUTURE WORK

This paper presented a reliability model which estimates life-cycle costs of systems that are remanufactured. These reliability-based, life-cycle costs can be used to compare design alternatives.

Contrary to many other system reliability models, this model describes repair during remanufacture or maintenance as leaving the system in neither same-as-new nor same-as-old states. Furthermore, this model accommodates system modification, in which failed parts are replaced with components with different failure characteristics. This feature portrays more accurately many instances of component replacement during remanufacture or maintenance. Replacement components may have different failure properties from the original components because of different suppliers of replacement parts and system upgrade or reconfiguration.

The model represents a population of systems as a collection of populations of the constituent parts. Part failure can result in replacement of the part with a component of the same or different type, or in replacement of the system. When only a portion of the system is replaced, the remaining parts of the system either remain unchanged or are reconfigured to accommodate the replacement part. The age distribution of each part population determines the failure characteristics of the corresponding part. Currently, this model describes series systems in which the components have densities of time to failure that can be represented by the two-parameter Weibull distribution.

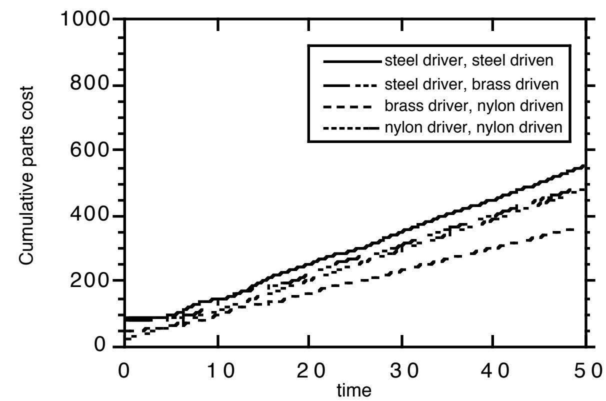

Fig. 12. Cumulative part replacement cost: No labor included

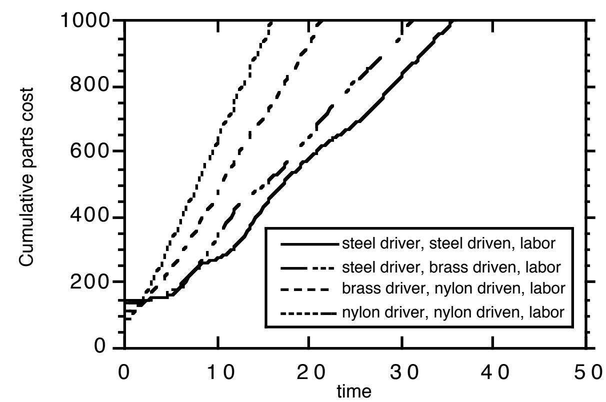

Fig. 13. Cumulative part replacement cost: Labor included

The basic model behavior simulates replacement of failed parts with components of the same type; this fundamental behavior was experimentally verified. Since it is common practice in remanufacture to replace failed parts with components of a different type, this situation was also modeled. Reliability theory necessary to predict system failure from the failure characteristics of the constituent parts in series was outlined. Finally, the model was applied to a mechanical series system to compare life-cycle costs of various combinations of part selection.

This model will be expanded to encompass systems with series, parallel, and standby subsystems, where component failure rates can be represented by a variety of distributions. Data from industries that perform remanufacture and maintenance will be used to select distributions and parameters for failure rates.

Integration of this model into a life-cycle cost optimization builds understanding of how part specifications and repair policies affect product life-cycle, and enables remanufacture and maintenance to become more cost-effective and viable. The increased viability of remanufacture will result in positive effects on both the environment and economy.

The stochastic nature of this method will complement a probabilistic design methodology that combines life-cycle and traditional design requirements (Wallace et al. 95). Life-cycle concerns project into the future and inherently involve uncertainty. Therefore, a probabilistic design framework that treats uncertainty in life-cycle factors such as future disassembly technologies and legislation, and uncertainty in traditional design parameters, such as material strength and costs, seems appropriate. This supports the long term goal to integrate life-cycle issues into a systems-oriented, computer-aided design tool so that consideration of environmental aspects will become an inherent part of the product design process.

ACKNOWLEDGMENTS

The author thanks the Eastman Kodak Company for the funding that made this work possible. The opportunity to collaborate with the Office Imaging Remanufacturing division formed the foundation of much of this work. I am also grateful to Arrow Automotive Industries and Nashua Cartridge Products for collaboration that provided many valuable insights about remanufacturing.

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