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Reliability Modeling in Design for Remanufacture

Remanufacture involves the disassembly, cleaning, and replacement or refurbishment of worn parts of products at the end of life. Since the essential goal of remanufacture is part reuse, the reliability of components is important. The goal of this work is to consider reliability effects on life-cycle costs to enable design for reuse. A reliability model is developed to better describe systems that undergo repairs performed during remanufacture or maintenance. This model allows replacement components to be of a type different from the original components. The behavior, preliminary experimental validation, and application of the model to an example are presented.

Motivation

In addition to resource conservation, design for product end-of-life is compelled by existing and impending product take-back laws that place product end-of-life responsibility on the manufacturer. Given this responsibility, the manufacturer may choose to pay increasingly higher fees for landfill or incineration, or have the product reused, repaired, remanufactured, or recycled for scrap material. While many design-for-end-of-life guidelines emphasize facilitating scrap-material recycling, significant resources are consumed during the recycling process. Furthermore, material degradation often results due to molecular breakdown and contamination, both of which are frequently characteristic of current recycling technologies. Individual product repair and maintenance are limited by the high labor costs that tend to cause discarding of repairable products. Lund (1983) observed that by recycling at the component instead of the material level, remanufacturing avoids the possibly unnecessary resource consumption of scrap-material recycling while preserving the value added to the component during manufacture. Also, the production-batch and off-site nature of remanufacturing results in a labor cost significantly lower than that required for individual repair.

While remanufacture is not suitable for all products, it is particularly appropriate for technologically stable items, where a large fraction of components can be reused. Product design that facilitates any of the steps of remanufacture, namely disassembly, sorting, cleaning, refurbishment, reassembly and testing, will facilitate remanufacture. However, the essential goal in remanufacture is part reuse. If a part cannot be reused as is or after refurbishment, the ease of disassembly, cleaning or reassembly will not matter. Refurbishment activities aim to return a part to a like-new or better condition, and include, for example, reboring out-of-round cylinders or fitting cylinders with sleeves.

When parts are to be reused, in either remanufacture or maintenance, the reliability of the part is very important. Collaboration was initiated with three companies that remanufacture a variety of products to learn about the remanufacture process and how products can be designed to facilitate remanufacture. These companies are Eastman Kodak, a manufacturer and remanufacturer of photocopiers, single-use cameras, and medical analysis equipment; Nashua Cartridge Products, a remanufacturer of toner cartridges; and Arrow Automotive Industries, a remanufacturer of automotive after-market parts. This collaboration offered insights on reliability issues across the companies and requirements for reliability modeling.

Existing reliability models are unsuitable for describing systems that undergo repairs performed during remanufacture. The goal of this research is to develop and validate reliability models to be applied in life-cycle cost estimations of systems where reuse of components is possible. These calculations may be used to explore initial part design and remanufacture process plan alternatives in the context of other life-cycle concerns.

Overview

This paper begins by highlighting related work in the fields of both life-cycle design and reliability. Selected reliability models with features closest to those desired are detailed before introducing the motivation for distinct characteristics incorporated into the model developed here. The goal of this paper is to illustrate properties of this model and demonstrate how it can be applied to compare design alternatives for mechanical series systems. This model currently describes series systems whose components have Weibull-distributed densities of time to failure. Therefore Weibull-related terminology and notation are defined. The model simulates the replacement of failed parts with components of either the same or a different type. Replacement parts can be either new or remanufactured. Components of the same type are those that have identical failure characteristics to the original component. The simulation results of replacement with the same type of parts were experimentally validated. Replacement of failed parts by components of a different type often more accurately portrays remanufacture, where the replacement component has different failure properties from those of the original part. The modification can be subtle, due to different sources for replacement components, or more drastic, due to reconfiguration of the system for upgrades or correction of known reliability problems. The interaction of multiple components in a system is described using series-system reliability theory. An example applies the model to compare the life-cycle costs of various combinations of mechanical elements and illustrates additional considerations for application to mechanical systems.

Literature Review

Researchers have considered the roles of failure and serviceability in life-cycle design. Gershenson and Ishii (1991) implemented Service Mode Analysis, which focuses on repair possibilities of various system malfunctions, in a computer tool that calculates serviceability indices of various user-defined failure

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phenomena. DiMarco et al. (1995) integrated Failure Modes and Effects Analysis into a computer-aided design tool to bring consideration of service costs early in the design process.

Ascher and Feingold (1984) surveyed the considerable body of reliability literature on repairable systems and observed that much of this work models one of two extremes. The first extreme represents the repair process as returning a system to a same-as-new condition. The other extreme, known as minimal repair or same-as-old, describes the system reliability after repair as identical to that before failure, which approximates repairs that involve the replacement of a small fraction of a system's parts. For this model to be exact, the replacement component must have the same distribution of time to failure as the original part, and the same age if the failure rate is age dependent.

Moderation of the above two extremes includes the following. Brown and Proschan (1980) model imperfect repair, where at each repair, renewal to the same-as-new state occurs with probability P and no age reduction or same-as-old occurs with probability 1-P. Nakagawa (1980) models partial renewal of a system at maintenance times, where the effective unit age is reduced to a proportion of the actual age. De la Mare (1979) fit Weibull distributions to data for successive times between failures for many types of systems. He used the estimated means in a cost model to optimize system life-cycle costs. Cozzolino (1968) developed two models, the n-component device model and the time accumulation model, that have some desirable properties which will underlie the unique features of the model developed here.

The n-component device model describes a system composed of n parts in series. The model tracks the ages of a system's constituent parts, and each part can have different distributions of time to failure. Each part's failure characteristics are also independent of other parts' failure processes. Time to first system failure is the minimum of the components' times to first failure. The device ages by accumulating time on its constituent parts, and the vector of component ages determines the density of future time to failure. Failure of one part results in replacement with a part of the same type. Since only the age of the replaced component is reset to zero while the other components retain their age, the system failure rate never returns to its initial value.

The time accumulation model, developed to reduce the complexity of the n-component device model, assumes that the n constituent parts are identical, so that the identity of the replaced part need not be tracked. At each failure, $1/n$th of the system accumulated age is lost.

Most models, including those of Cozzolino, do not allow for the possibility of system modification by limiting replacement parts to those with the same failure density function as the original parts. A main differentiating feature in the model developed here is that replacement parts can have failure density functions different from the original parts. This additional capability is motivated by common practices in remanufacture.

Characteristics of Model

This section will present the model developed here to describe systems that undergo repairs performed during remanufacture or maintenance. First, the terminology (Jiang et al., 1998) and notation used in the model are delineated.

- Part or component. Item that is not subject to further disassembly. Upon failure, it is replaced by a part of the same or different type.
- Socket. Circuit or equipment position in a system that holds a part of a given type.
- Series system. System structured such that the failure of any one of the system's constituent parts results in system failure.
- System population. Collection of systems with the same combination of parts.
- Part population. Collection of parts that hold the same socket position in the system population.
- N. Number of parts or components that comprise a system.
- x. Age of part.
- $f(x)$. Failure density function.
- $F(x)$. Failure probability function.
- t. Time elapsed since a population of parts was put in service.
- $q_i(t_n)$. Age distribution, or fraction of population of parts at time t_n with age $i\Delta t$, $i = 0, 1, \ldots, n$.
- $q_0(t_n)$. Fraction of population of parts at time t_n with age 0, representing parts that have just been put into service, and for time $t_n > 0$, as replacements for failed parts.
- $\bar{a}(t_n)$. Average age of a population at time t_n.
- Population replacement rate. The fraction of the population replaced per unit time.

The model developed here describes a population of N-component series systems. The N parts have independent and different distributions of time to failure. The population of N-component systems is represented as a collection of N populations of constituent components, and parts are treated as members of their respective populations.

The basic assumptions of the model are:

1. The size of the system population is constant.
2. The N parts have independent and possibly different failure density functions.
3. Part failure results in replacement of the part with a part of the same or different type.

The relationships of these assumptions to actual populations are as follows. Actual population sizes may not be constant. For example, some systems may not be replaced after failure, while other systems may be added after the initial population was put into service. The current model assumes that each failed system is replaced and that no additional systems are added. The second assumption allows each of the system's constituent parts to have different failure density functions, thus making the model applicable to more systems than one that assumes that all constituent parts have the same failure density function. However, the failure characteristics of the constituent parts are assumed to be independent, i.e., failure of one part does not affect failure of other parts in the system.

Finally, this model allows replacement parts to have failure density functions different from the original parts. This additional capability allows the model to describe more accurately common practices in remanufacture. For example, bearings are often replaced with higher-durability bearings during remanufacture. Many refurbishment processes change reliability characteristics by altering the system configuration. For example, bronze bushings are installed in distributor housings that wore due to the lack of separate bearings in the original design.

Also, parts identical to those installed during original manufacture may effectively have different failure characteristics when installed later in the life of a product. This may be due to the different working conditions of remanufacture or maintenance from those of original manufacture, which may increase or decrease, for instance, likelihood of human error. The effects on reliability of mistakes made during manufacture or remanufacture that cannot be modeled by a probability distribution are beyond the scope of this probabilistic reliability model.

The age distributions of each of the part populations are tracked to determine the reliability of the composite system population. Failure density and age distributions associated with each part population are used to calculate the probability of failure of that part at any given time. Failure of a part may result either in part replacement as described above or in the

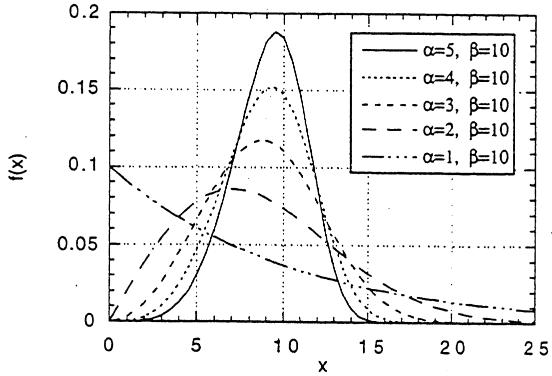

Fig. 1*a* **Weibull probability distributions**

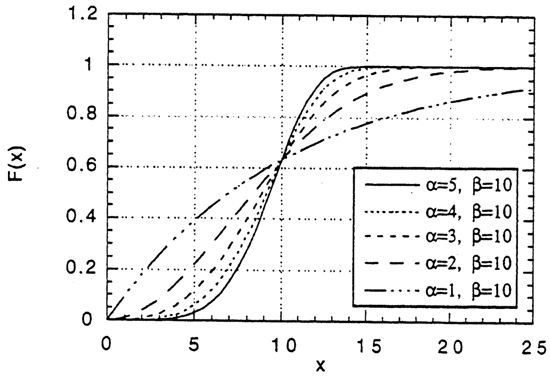

Fig. 1*b* **Weibull probability of failure**

replacement of the entire system with either an identical or a different system.

In this model, the repair policy determines actions executed upon part failure. Corporate refurbishment policy significantly affects both the system reliability and the consequent remanufacture cost of a given original design. Some companies may choose to always replace a particular component without inspection, due to product reconfiguration or past reliability problems, while others will replace based on either actual component failure or projected remaining life. Also, some parts, such as those that are consumed during use, will always be replaced. These policies can be accommodated in the model developed here.

This model describes series systems where the density of time to failure of each component is represented by the two-parameter Weibull distribution. The extension of this model to use other distributions is fairly straightforward. The Weibull distribution was selected because it is appropriate for many engineering applications.

Weibull Distribution Notation

The Weibull probability density function is:

$$f(x) = \frac{\alpha x^{\alpha-1}}{\beta^{\alpha}} \exp\left[-\left(\frac{x}{\beta}\right)^{\alpha}\right], \quad 0 \le x \le \infty \qquad (1)$$

Figure 1(a) plots Weibull distributions with $\beta = 10$ and values of α from 1 to 5. Alpha is the shape parameter; a larger α reduces spread about the expected value. Beta is the scale parameter; as α increases, the peak of the distribution approaches β. A component with a Weibull-distributed density of time to failure has an expected lifetime of approximately β, and precisely $\beta\Gamma[1/\alpha + 1]$. An α value of 1 yields the exponential distribution, and α equal to 2, the Rayleigh distribution. The effects of α and β on the model output will appear throughout the paper.

The value of $f(x)$ is the probability that a part fails between times x and $x + dx$. $F(x)$, the integral of $f(x)$ from 0 to x, represents the probability that a component fails at any time up to x.

$$F(x) = \int_0^x f(x)dx \qquad (2)$$

Figure 1(b) plots $F(x)$ curves corresponding to the densities of Fig. 1(a). Note that β locates the intersection of the family of integrals. 1-$F(x)$ is the probability that the part will survive past x. Another quantity often used in reliability is the failure rate function which is represented by:

$$\lambda(x) = \frac{f(x)}{1 - F(x)} \qquad (3)$$

The failure rate is the conditional probability of failure at x given survival to x. Note that the failure rate becomes greater than 1, and that the failure rate function is not a density function.

Simulation

The density function of time to failure of every component in a system is used to calculate the part replacement requirements for a population of systems. An age distribution obtained at each time step for each part population determines failure rates for the following time step. The failure rates determine the replacement-part-cost portion of the system life-cycle cost. Failed parts can be replaced with components of either the same or different type. First presented will be the simulation of replacing failed parts by the same type of components. The preliminary experimental validation of this basic model behavior is described. Next presented is the simulation of replacing failed parts with components of a different type. The behavior of the model is introduced using populations of "single-component systems." Subsequent sections will describe how the interactions between multiple components of a system are treated.

Description of Basic Simulation. Age bins are used to track the age distribution of a population of parts. The time-to-failure density determines the portion of the contents of each bin that survive to the next time step, appearing as contents for the next older bin, and the portion that fails. The portion that fails is replaced and represented as contents in the zero-age bin. Figure 2(a) through 2(c) track the age bin distributions for three consecutive time steps for a population of parts whose time-to-failure Weibull density corresponds to $\alpha = 2$, $\beta = 3$. Age bins are created at increments equal to the time between events, e.g., number of years between remanufacture or maintenance activities. Time between events, or bin size, of 1 was used to produce the results shown in Figs. 2(a) through 2(c).

Initially, all parts are in the first bin as shown in Fig. 2(a): the population consists only of new components. That is, at $t_0 = 0$, $q_0(t_0) = 1$, where q_i is the fraction of parts in the ith bin.

At the next time step, the failure density is integrated using numerical methods developed by Senin et al. (1996) from zero to one time increment to find the probability of failure. The portion of the population that survives advances to the next age bin, and the portion that fails is replaced and represented as items in the first age bin, as shown in Fig. 2(b). That is, at $t_1 = \Delta t$, the fractions of the first two bins become:

$$q_1(t_1) = q_0(t_0)\left[1 - \int_0^{\Delta t} f(x)dx\right]$$

$$q_0(t_1) = q_0(t_0)\left[\int_0^{\Delta t} f(x)dx\right] \qquad (4)$$

Again, portions of both age bins survive and advance to the next age bin, and portions of failed parts from both bins appear

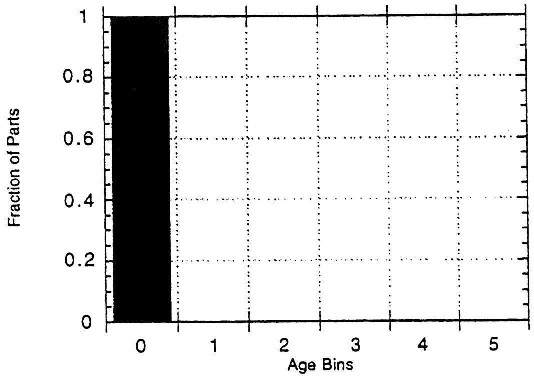

Fig. 2a Bin distribution at $t_0 = 0$

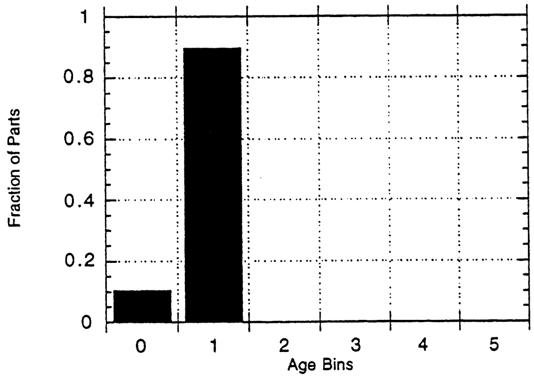

Fig. 2b Bin distribution at $t_1 = \Delta t$

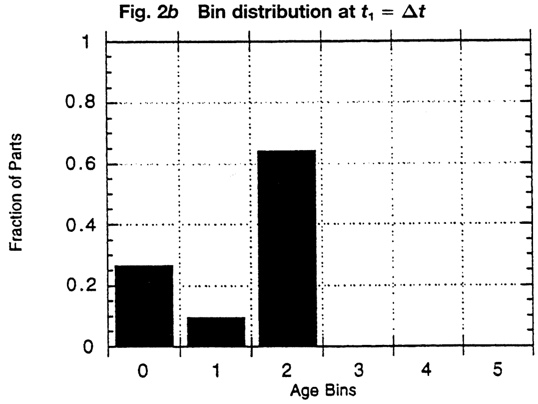

Fig. 2c Bin distribution at $t_2 = 2\Delta t$

as replaced parts in the first bin. The proportions of each bin for $t_2 = 2\Delta t$, the following time step, are calculated as:

$$q_2(t_2) = q_0(t_0)\left[1 - \int_0^{2\Delta t} f(x)dx\right]$$

$$q_1(t_2) = q_0(t_1)\left[1 - \int_0^{\Delta t} f(x)dx\right]$$

$$q_0(t_2) = q_0(t_0)\left[\int_{\Delta t}^{2\Delta t} f(x)dx\right] + q_0(t_1)\left[\int_0^{\Delta t} f(x)dx\right] \quad (5)$$

At $t_n = n\Delta t$, the fractions of parts in each bin are:

$$q_n(t_n) = q_0(t_0)\left[1 - \int_0^{n\Delta t} f(x)dx\right]$$

$$q_{n-1}(t_n) = q_0(t_1)\left[1 - \int_0^{(n-1)\Delta t} f(x)dx\right]$$

$$\dots$$

$$q_2(t_n) = q_0(t_{n-2})\left[1 - \int_0^{2\Delta t} f(x)dx\right]$$

$$q_1(t_n) = q_0(t_{n-1})\left[1 - \int_0^{\Delta t} f(x)dx\right]$$

$$q_0(t_n) = q_0(t_0)\left[\int_{(n-1)\Delta t}^{n\Delta t} f(x)dx\right] + q_0(t_1)\left[\int_{(n-2)\Delta t}^{(n-1)\Delta t} f(x)dx\right] + \dots + q_0(t_{n-2})\left[\int_{\Delta t}^{2\Delta t} f(x)dx\right] + q_0(t_{n-1})\left[\int_0^{\Delta t} f(x)dx\right] \quad (6)$$

The average age of the population is calculated by summing over all the age bins the product of the fraction of parts in that bin and the age of the bin:

$$\bar{a}(t_n) = \sum_{i=0}^{n} q_i(t_n) i\Delta t \quad (7)$$

Results of Basic Simulation. Figure 3(a) plots the average age of constant-size populations of identical parts that are replaced by components of the same type upon failure. Each curve represents a population of parts with a particular Weibull distribution of time to failure. The plots shown correspond to a constant value of β equal to 10 paired with α values of 1, 2, 3, 5 and 10. Both the horizontal and vertical axes have the same units of time, e.g., minutes, hours, or years.

Several characteristics of Fig. 3(a) are of interest. First, the average age eventually reaches steady state. This agrees with Drenick's Theorem (Drenick 1960), which states that the superposition of an infinite number of independent equilibrium re-

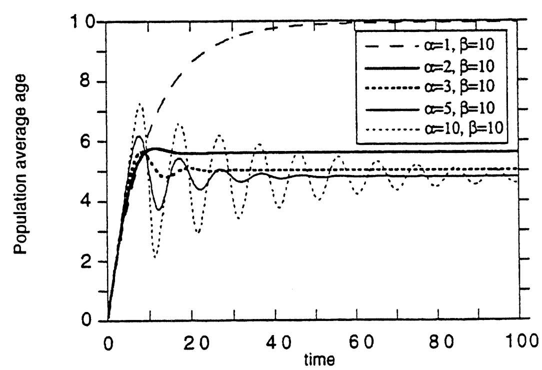

Fig. 3a Population average age

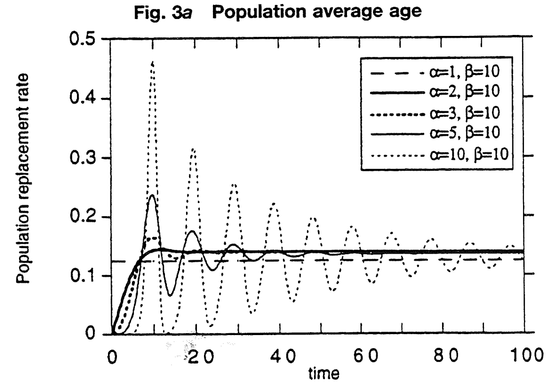

Fig. 3b Population replacement rate

newal processes is a homogeneous Poisson process. A homogeneous Poisson process is one that can be represented by an exponential distribution, which has a constant failure rate. A population with a constant failure rate and part renewal upon failure has a constant average age. The value of the steady state age depends on Weibull parameters α and β. The dependence on β is not surprising; higher values of β for a given set of α's yield higher values for expected time to failure and thus average age.

Alpha affects both the steady state value and the degree of oscillation. Recall from Figs. 1(*a*) and 1(*b*) that as α increases, the window of time during which a majority of parts fail decreases. For high values of α, very few parts will fail until time β, at which time almost all the parts will fail immediately. During the low-failure period, the average age will increase monotonically. Then as increasingly large numbers of parts fail, the replacement of a significant portion of the population causes the average age to drop until the wave of failure is over. The newly installed base of parts then ages steadily until the next failure wave. During each oscillation, a number of parts fail outside the time window during which most of the population fail. The population thus becomes more age-diversified with each cycle, and the oscillations in average age diminish. The higher the value of α, the fewer parts fail outside the tighter expected failure period, and thus the greater the oscillations in average age and the longer it takes for age diversification to occur. As α increases, the mean of the average age approaches $\beta/2$. This is intuitive when one considers the upper bound as α approaches infinity. Physically, such a distribution of time to failure implies that no parts fail until time β, at which time all the parts fail. Therefore this population would have a saw-toothed average age plot that does not decay and is bounded between 0 and β. Note that for α equal to 1, the average age approaches β, as opposed to $\beta/2$, the corresponding value for increasing α. Physically, this higher asymptotic value can be explained by the presence of more "very old" parts, corresponding to the exponential distribution of time to failure shown in Fig. 1(*a*).

Figure 3(*b*) plots the population replacement rate corresponding to Fig. 3(*a*). The trends of Fig. 3(*b*) are consistent with those of Fig. 3(*a*). Average age decreases as part replacement increases since more parts are being replaced by new, or zero-age parts. In continuing development of this model, steady state values for replacement rate were determined by Jiang et al. (1998) to depend on expected life and time interval between replacements.

Experimental Validation of Basic Simulation. An experiment was performed to validate the basic behavior of the model. This experiment applied the model to a fastening system and involved obtaining data on the number of disassembly and reassembly cycles before a screw strips a hole in plastic.

For the experiment, a grid of holes was drilled in a sheet of polypropylene. Thread-forming screws were repeatedly inserted into the holes and removed using a power screwdriver at a constant torque until the screw continued to spin when fully inserted.

The number of rows of holes represents the number of systems in the sample. A sample size of 50 systems was used. When a hole fails, 'part replacement' involves using the next hole in the same row. A screw removal-and-insertion cycle performed on the sample constitutes a time step. The number of screw removal-and-insertion cycles until failure was recorded for each hole. This was used to obtain a distribution of number of cycles-to-failure for the sample. The number of cycles survived by each active hole averaged over the sample at each time step yields the average age plot. The data associated with the final holes were not used to obtain the cycles-to-failure distribution because those holes had not failed yet, but they were used to calculate the average age.

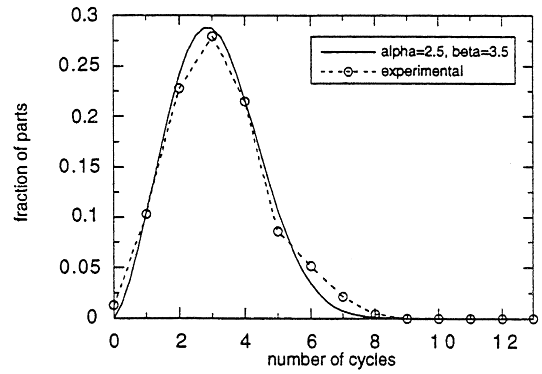

Fig. 4a **Sample cycles-to-failure histogram vs. Weibull distribution with** $\alpha = 2.5$, $\beta = 3.5$

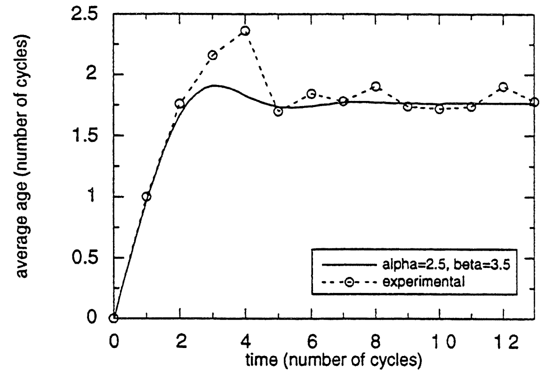

Fig. 4b **Average age obtained experimentally vs. through simulation using Weibull distribution** $\alpha = 2.5$, $\beta = 3.5$

Figure 4(*a*) compares the sample histogram of cycles to hole failure with the Weibull distribution that produced the least-squared error between the experimental data points and the values determined by the distribution. Figure 4(*b*) compares the average age yielded experimentally with that produced through simulation using the above Weibull distribution.

In a second experiment, a new torque was used to obtain a different distribution of cycles-to-failure of the holes. The new torque resulted in a less controlled and predictable process leading to a lower value of α. Recall from Fig. 3(*a*) that when α is closer to 1, for similar values of β, the asymptotic average age is higher than for higher values of α.

Figure 5(*a*) compares the sample histogram of cycles to hole failure for the new torque with the Weibull distribution producing the least squared error. Figure 5(*b*) compares the experimental average age with the average age simulated using the corresponding Weibull distribution.

For both experiments, the agreement is reasonable considering the relatively small sample size of 50. The smaller sample size is much more sensitive to outliers and thus displays a greater noise level than if a larger sample were used.

Simulation of System Modification. The preceding simulation results and experimental validation were for the replacement of failed parts with components of the same type. As noted earlier, repairs made during remanufacture often change the reliability characteristics of a system by replacing failed parts with components of a different type. The remaining parts of the system can either stay the same or be reconfigured to accommodate the replacement component. Similar experimental validation of this behavior could involve using, in the first hole of each row, screws with a thread density that is different from the screws used in the remaining holes of each row. The different thread density will result in a different distribution of disassembly and reassembly cycles-to-failure for identical holes. In the remanufacture of toner cartridges, when a plastic boss is stripped, a larger or coarser-thread screw is often used in place of the original screw.

The simulation results for the replacement of failed parts with different components are shown in Figs. 6(a) and 6(b). Subsequent failure of replacement components results in replacement by the same components, i.e., parts of the original type are not reintroduced into the population.

Figure 6(a) and 6(b) chart the replacement of an initial population of parts with Weibull parameters $\alpha = 3$, $\beta = 10$, denoted (3, 10), with components of Weibull parameters $\alpha = 10$, $\beta = 10$, denoted (10, 10). Subsequent replacement of failed (10, 10) components are with the same (10, 10) components. For reference, replacement of an initial population of (3, 10) parts by the same (3, 10) parts and replacement of an initial population of (10, 10) components by the same (10, 10) components are also plotted. Of interest in the average-age and part-replacement plots are the phase shift and reduced oscillation of the (3, 10)-to-(10, 10) curve relative to the (10, 10)-to-(10, 10) curve. An original population of (3, 10) parts fail earlier and with more spread between time-of-failures than an original population of (10, 10) components. Therefore, the first replacement batch of (10, 10) components appear earlier and more staggered over time for a population that began with (3, 10) parts than for a population that began with (10, 10) components. The effect of this initial difference carries over to subsequent replacement cycles.

Figure 6(a) and 6(b) suggest that a possible strategy to reduce oscillation in replacement part demand from manufacture or remanufacture is to install an initial population of parts that has a lower value of α, corresponding to a greater spread in time to failure. When the initial parts fail, they can be replaced with parts with a higher value of α, so that subsequent failure of individual systems can be more precisely predicted. However, this is likely to be unnecessary for products that are put into service at different times, thereby resulting in offset times of failure. The predictable peaks in failure can be beneficial from a maintenance point of view. It may be more efficient to perform scheduled replacement of parts for entire populations of systems that are close to failure than on an individual basis as the parts fail.

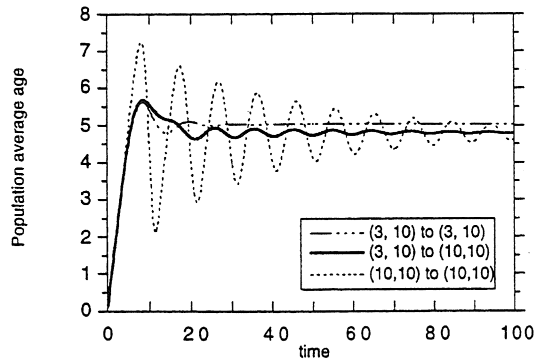

Fig. 6a Population average age

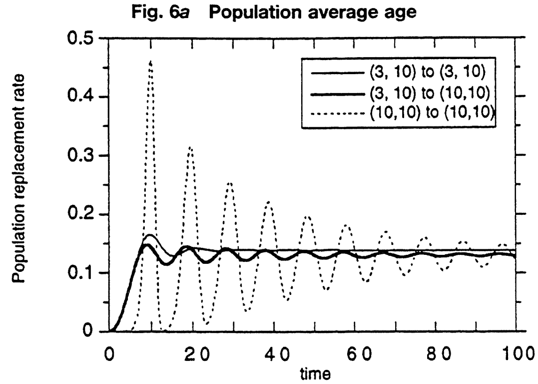

Fig. 6b Population replacement rate

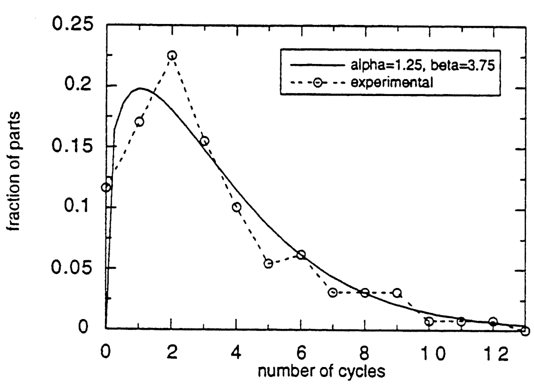

Fig. 5a Sample cycles-to-failure histogram vs. Weibull distribution with $\alpha = 1.25$, $\beta = 3.75$

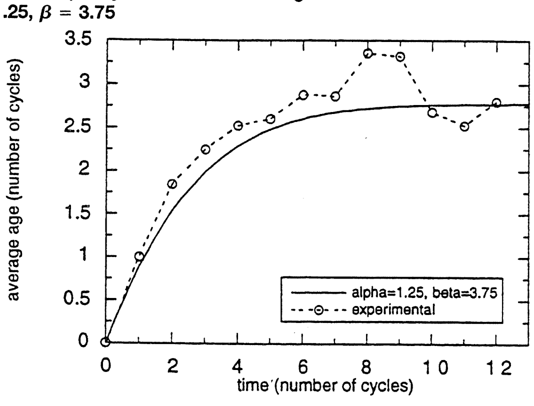

Fig. 5b Average age obtained experimentally vs. through simulation using Weibull distribution with $\alpha = 1.25$, $\beta = 3.75$

Series System Behavior

The previous sections described the behavior of the model for single populations of components. This section will illustrate how the reliability of a system is obtained from the reliability of the constituent parts in series. In a series system, the failure of any one of the constituent parts results in system failure.

The failure rate of a series system is the sum of the failure rates of the components:

$$\lambda_{sys}(x) = \sum_{i=1}^{N} \lambda_i(x) \tag{8}$$

The reliability of a series system is the product of the reliability of the components:

$$R_{sys}(x) = 1 - F_{sys}(x) = \prod_{i=1}^{N} 1 - F_i(x) \tag{9}$$

From (3), the failure density of a series system whose parts have Weibull failure densities is:

$$f_{sys}(x) = \lambda_{sys}(x) R_{sys}(x) = \sum_{i=1}^{N} \frac{\alpha_i x^{\alpha_i - 1}}{\beta_i^{\alpha_i}} \prod_{i=1}^{N} e^{-(x/\beta_i)^{\alpha_i}}$$

$$f_{sys}(x) = \sum_{i=1}^{N} \frac{\alpha_i x^{\alpha_i - 1}}{\beta_i^{\alpha_i}} (e^{-\Sigma_{i=1}^{N} (x/\beta_i)^{\alpha_i}}) \tag{10}$$

System failure can result in either partial or complete replacement of the system. Components of a system are sometimes arranged or joined in a manner that requires the replacement of more than one part upon the failure of a single part. Also,

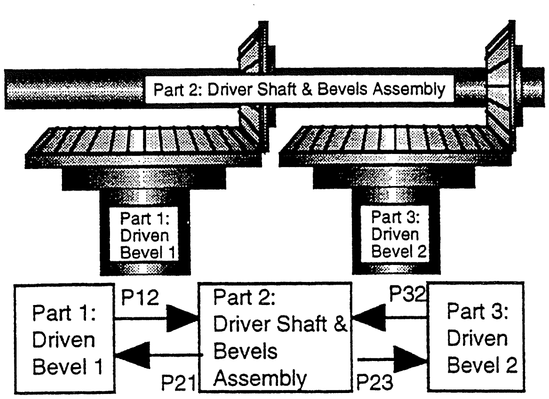

Fig. 7 Failure characteristic representation for mechanical elements in series

part consolidation often results in single components containing multiple features, the failure of any one of which would require component replacement. A possible product design strategy would be making failure-prone features separable, so that the failure of a small portion of a component does not require the replacement of a largely unaffected and possibly expensive part (Shu and Flowers, 1995).

Series Mechanical Systems

This section presents additional considerations for application of the model to mechanical systems. The model is then applied to an example mechanical system to compare life-cycle part-replacement costs for various combinations of component selection.

Wear and failure of mechanical components often occur due to relative motion between parts, and thus the reliability of many mechanical components depends on the interaction with the component with which it is coupled. For example, a gear may have different failure characteristics depending on the gear with which it meshes. Therefore, failure characteristics can be defined as interactions between components.

For example, consider the driver-shaft-and-bevels assembly and driven bevel pinions illustrated in Fig. 7. The interactions between the driver and driven bevels, as well as the material and geometry characteristics of each gear, determine the gear failure parameters. Tables 1 and 2 contain hypothetical gear costs and failure characteristics as α and β of the Weibull-distributed time-to-failure density. Table 2 contains failure density parameters of driver-assembly and driven bevels made from three different materials as a function of the material of the meshing gear. The trend assumed by the failure parameters is that a softer material wears faster when meshed with a harder material. A sensitivity analysis on this hypothetical data has not been performed.

If the entire driver assembly is replaced as a unit upon failure of either driver bevel, the assembly has two simultaneous interactions. The resultant failure density of the assembly due to the failure of either driver bevel can be found using (10).

Table 1 Gear costs used in simulation

Gear material	Driven Bevel 1	Driver shaft/bevel Assembly	Driven Bevel 2
Polished steel	20	50	20
Brass	15	40	15
Nylon	5	15	5

Table 2 Failure distribution of bevels for various material combinations

Bevel Material	Bevel	*Meshing Bevel Material*		
		Polished steel	*Brass*	*Nylon*
Polished steel	Driver	α=6, β=8	α=7, β=12	α=8, β=16
	Driven	α=6, β=16	α=7, β=24	α=8, β=32
Brass	Driver	α=3, β=4	α=4, β=8	α=5, β=12
	Driven	α=3, β=8	α=4, β=16	α=5, β=24
Nylon	Driver	α=1, β=1	α=2, β=2	α=2, β=3
	Driven	α=1, β=2	α=2, β= 4	α=2, β=6

The part cost and failure data of Tables 1 and 2 are used to compare life-cycle part-replacement costs for four combinations of component selection. These combinations are: steel driver-assembly bevels with steel driven bevels, steel driver-assembly bevels with brass driven bevels, brass driver bevels with nylon driven bevels, and nylon driver bevels with nylon driven bevels. In each combination, both the driver bevels are of the same material, as are both the driven bevels.

Several simplifications over typical practice are made. The effects of the attachment between the bevels and the shafts are neglected. Meshing gears are usually both replaced when either needs to be replaced, but here only the failed part is replaced, and the failure characteristics of one gear are assumed to be independent of the meshing gear age. The driver-shaft-and-bevels assembly is counted as one component and replaced as a unit.

The cumulative part replacement cost is calculated by summing the product of parts replaced and part replacement cost over the time the population was in service. The cumulative part costs for the above material combinations, shown in Fig. 8(*a*), suggest that the use of cheaper components is more cost

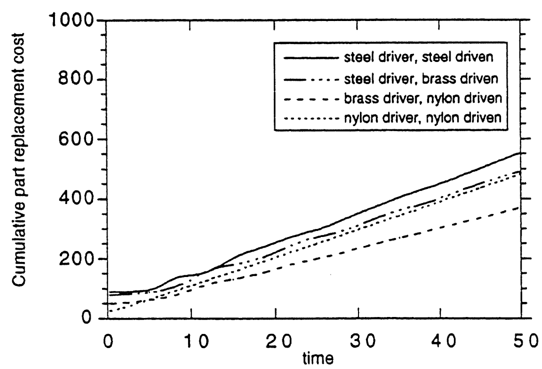

Fig. 8*a* Cumulative part replacement cost: no labor included

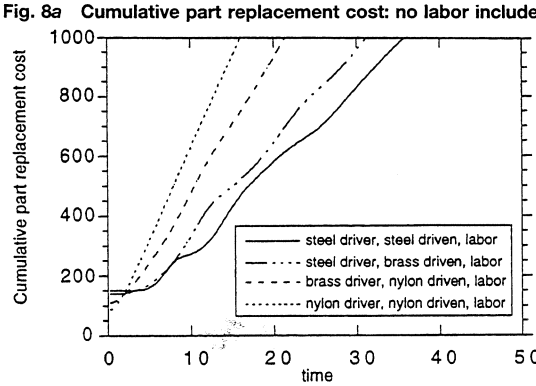

Fig. 8*b* Cumulative part replacement cost: labor included

effective. However, the cumulative cost included only component costs, not labor cost, nor the cost of disruption while the failed part is being replaced. Figure 8(*b*) plots the total population part-replacement costs obtained by adding a uniform cost of 60 to the part costs in Table 1. This additional cost can represent either a labor cost or other cost that is incurred each time a part is installed or replaced. The results are then reversed: the most cost effective combinations are those that incur a larger part cost, but also last longer. This confirms that the labor and disruption costs associated with replacing a component, in addition to the cost of the component, should determine component selection.

Summary and Future Work

This paper presented a reliability model which can be used to estimate life-cycle costs of systems that are remanufactured. These reliability-based, life-cycle costs can be used to compare design alternatives.

Contrary to many other system reliability models, this model describes repair during remanufacture or maintenance as leaving the system in neither same-as-new nor same-as-old states. Furthermore, this model accommodates system modification, in which failed parts are replaced with components with different failure characteristics. This feature more accurately portrays many instances of component replacement during remanufacture or maintenance. Replacement components may have different failure properties from the original components because of different suppliers of replacement parts, system upgrade or reconfiguration, or installation conditions during remanufacture or maintenance different from original manufacture.

The model represents a population of systems as a collection of populations of the constituent components. Part failure can result in replacement of the part with a component of the same or different type, or in replacement of the system. When only a portion of the system is replaced, the remaining parts of the system either remain unchanged or are reconfigured to accommodate the replacement component. The age distribution of each part population determines the failure characteristics of the corresponding part. Currently, this model describes series systems in which the components have densities of time to failure that can be represented by the two-parameter Weibull distribution.

The basic model behavior simulates replacement of failed parts with components of the same type; this fundamental behavior was experimentally validated. Since it is common practice in remanufacture to replace failed parts with components of a different type, the model developed is capable of describing this situation. Reliability theory necessary to predict system failure from the failure characteristics of the constituent parts in series was outlined. Finally, the model was applied to a mechanical series system to compare life-cycle costs of various combinations of component selection.

This model will be expanded to encompass systems with series, parallel, and standby subsystems, where component failure rates can be represented by a variety of distributions. Data from industries that perform remanufacture and maintenance will be used to select distributions and parameters for failure rates to further validate and develop this model.

Integration of this model into a life-cycle cost optimization builds understanding of how component specifications and repair policies affect product life-cycle, and enables remanufacture and maintenance to become more cost-effective and viable. The increased viability of remanufacture may result in positive effects on both the environment and economy.

The stochastic nature of this method will complement a probabilistic design methodology that combines life-cycle and traditional design requirements (Wallace et al., 1996). Life-cycle concerns project into the future and inherently involve uncertainty. Therefore, a probabilistic design framework that treats uncertainty in life-cycle factors such as future disassembly technologies and legislation, and uncertainty in traditional design parameters, such as material strength and costs, seems appropriate. This supports the long term goal to integrate life-cycle issues into a systems-oriented, computer-aided design tool so that consideration of environmental aspects will become an inherent part of the product design process.

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