

## Modeling and Simulation of Products' Life Cycles Using Petri Nets\*

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### 1. Introduction

Existing and impending legislation has urged many manufacturers to recycle their products. Under the legislation, manufacturers introduced DfD (Design for Disassembly) and DfR (Design for Recycling) approaches<sup>1)2)</sup> to the development of their products. This has brought about the recycling of some types of products at end-of-life phases. The wastes discharged in some other phases (e.g., production and distribution phases) have been also recycled. In such cases, the streams of objects (i.e., products, subassemblies, parts, raw materials, regenerated materials, wastes and so on) constitute many recycling cycles of objects, and the quantities of the objects in the streams fluctuate according to the fluctuation of the products' demands.

The aim of our study is to simulate the fluctuating streams throughout the products' life cycles including material production, part production, assembly, recycling, and so on. Through the simulation, we can see whether the objects are recycled without being stockpiled. We can also evaluate the effectiveness of recycling as a result of the simulation. In this paper, we propose a method of modeling and simulating the streams of objects, which is based on colored petri nets<sup>3)</sup>.

Nonomura et al.<sup>4)</sup> also proposed life-cycle simulation which can evaluate products' life cycles from economical and environmental viewpoints. So far our approach has not considered the economical viewpoint, however our approach can evaluate life cycles from the viewpoint of the smoothness of objects' streams as well as the viewpoints of the reduction of raw material usage and wastes. Our approach also has a characteristic that models can be made and viewed easily and visually, which is brought by using the colored petri nets.

### 2. Life cycle models of products

#### 2.1 Representation of life cycles using colored petri nets

We use colored petri nets for modeling life cycles. Figure 1 shows an example of the petri-net representation. "Object Stream model (OS model)" and "Stream Control model (SC model)" represent the streams of objects and the control of the streams respectively. These models are combined, and we call the combined one the "life-cycle model". This model consists of the following components.

- Places: Each place expresses the state or type of an

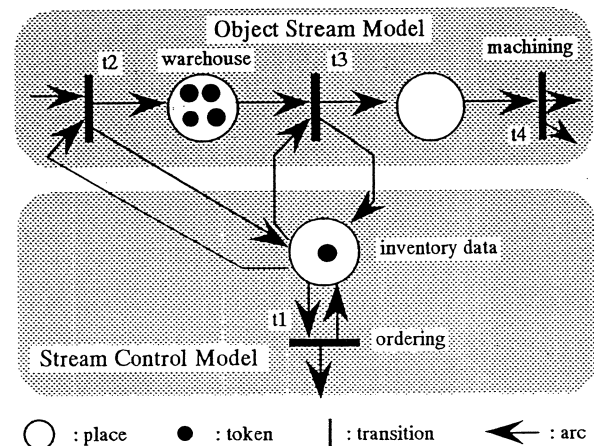


Fig. 1 A part of the life-cycle model

object or data. For example, the place, "warehouse", in Fig. 1 represents the state that a part is in the warehouse, and the place, "inventory data", indicates that the data are for the inventory of the warehouse.

- Tokens: Each token expresses an object or data. If the object is countable, one token corresponds to one object. Otherwise (i.e., the object is uncountable), the place can take only one token that represents the total amount of object and its quantity is described in the token as its attribute. If the token expresses the data, the place can also take only one token and the data are described in it as its attributes.
- Transitions: Each transition expresses the state transition of objects or the transaction of data. Occurrence of the state transition or performing the transaction is called "firing the transition". The condition and time period for firing are described in each transition. If the condition is satisfied, the transition is fired for the period.
- Arcs: Input arcs pointing from some places to a transition express that the objects or data in the places are required for firing the transition. Output arcs pointing from a transition to some places express that the objects or data in the places are generated or changed as the result of firing.

#### 2.2 Firing transitions

We can perform simulation using the life-cycle model by firing transitions in succession. At each time in the simulation, it is checked whether the condition described in each transition is satisfied by using the attributes of the tokens in the places that are connected with the transition by the input arcs. If the condition is satisfied, the tokens are

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removed and the transition is fired for the period. When firing is finished, the new tokens are generated in the places that are connected with the transition by the output arcs. The way to determine the attributes of the new tokens is also described in the transition.

In the example shown in Fig. 1, the token for "inventory data" has two attributes: the number of parts in the warehouse,  $a$ ; and the number of the parts that were ordered the last time and have been received by that time,  $b$ . The transition  $t1$ , which represents ordering a certain quantity of

the parts, checks these attributes to determine if its condition is satisfied. The condition described in transition  $t1$  is whether  $a$  is less than a lower limit and all the parts ordered the last time have been received by that time. If this condition is satisfied,  $t1$  is fired and the token in "inventory data" is removed. Then the new token whose attribute  $a$  is not changed but whose attribute  $b$  becomes 0 is generated in "inventory data". The attributes  $a$  and  $b$  are also modified when the transitions  $t2$  and  $t3$  are fired, which represent the delivery to and from the warehouse respectively.

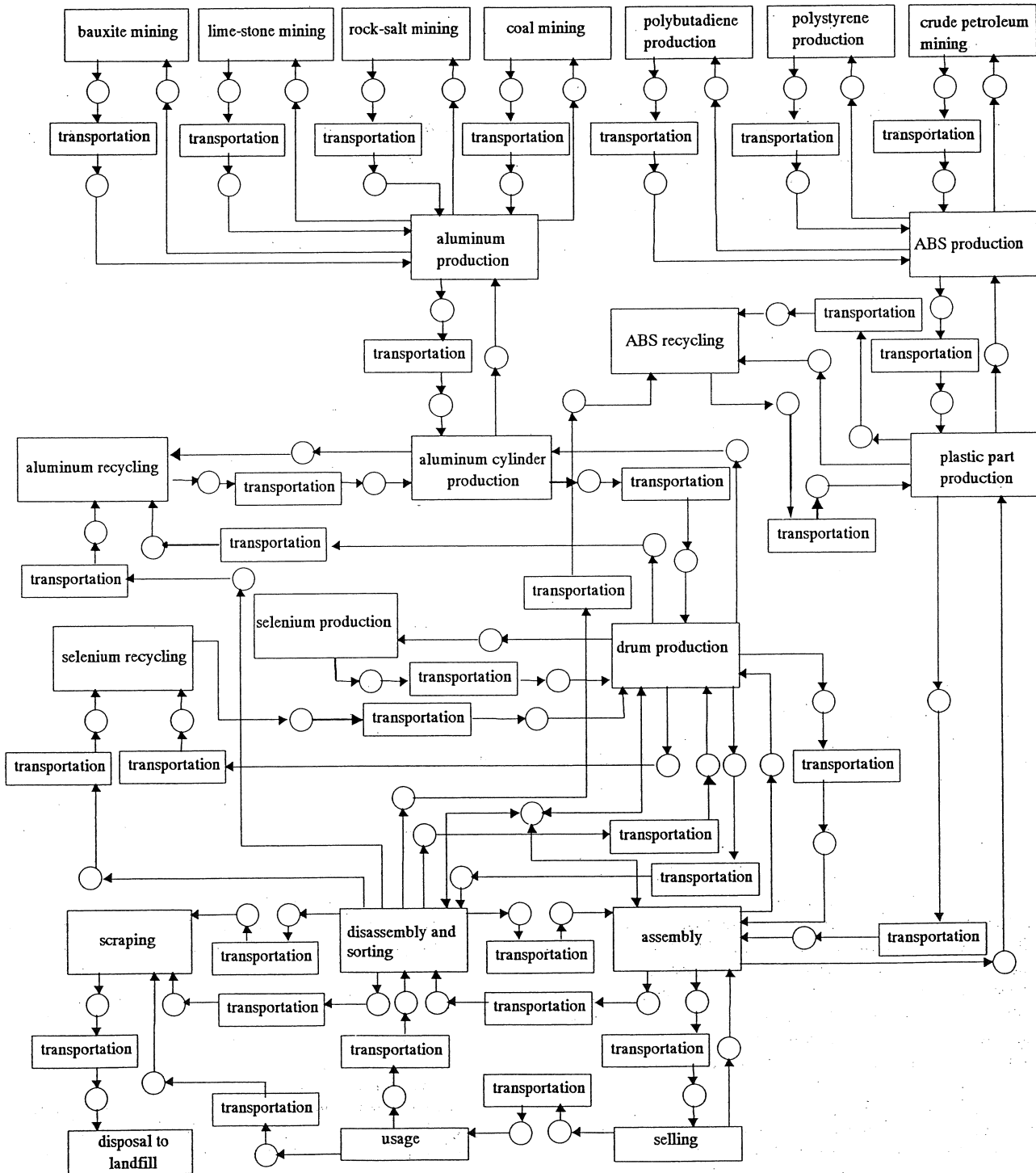


Fig. 2 Life-cycle model of the copier drum.

### 2.3 Attributes of parts and products

The tokens representing parts and products have several attributes which play important roles during the simulation. Each token representing a part has the attribute of the time to failure. We give this attribute by generating exponential random numbers when we generate the token representing a new part. The exponential distributions for generating the exponential random numbers differ from one kind of part to another. If the part has been disposed but will be reused, the token of the part has also the attribute of the period that the part has been used for.

Each token representing a product has three attributes: the time to failure; list of the parts composing it; and the time when the product usage started. We can calculate the time to failure using the data on the times to failure of the parts. We consider parallel, series, and m/n (m out of n) types of products to calculate the time to failure of the products.

These attributes are used mainly for the following judgements in the simulation:

- judgement whether a product or part is to be debugged because of its bad quality, in the manufacturing stage
- judgement whether a product fails, in the usage stage
- judgement whether a disposed product or part can be reused or recycled, at the end-of-life stage

### 2.4 Generation of orders

The products are ordered by firing a so-called source that is a transition having no input arcs. Time between firing depends on an exponential distribution and its probability density function is:

$$f(t) = -or \cdot \exp(-or \cdot t) \quad (1)$$

where  $or$  is an order rate defined as the order number for a certain period (e.g., month).<sup>9</sup> We give the order rate of each period before the simulation, and determine the time between firing by generating exponential random numbers in the simulation.

## 3. Modeling and simulation for a copier drum

We carried out the modeling and simulation for a copier drum. This drum is made of aluminum, selenium, and ABS. We built three life-cycle models of it corresponding to the following three cases respectively.

Case 1: all the used drums are discarded.

Case 2: the used drums are discarded or material-recycled.

Case 3: the used drums are discarded, material-recycled, or reused.

We built these models on the following assumptions:

- We deal with closed-loop recycling in which regenerated materials and reusable parts are used for the same kind of copier drums.
- In material recycling, we mix the regenerated material with the virgin one in a certain ratio to produce the material used in new parts. Each kind of material has its own ratio.
- The wastes discharged in production stages, such as chips of materials, are recycled in each case.
- The products are used until they fail.
- In Case 2 and 3, if the usage period of a product is very long, the quality of materials used in the product is regarded as deteriorated and the product is discarded.

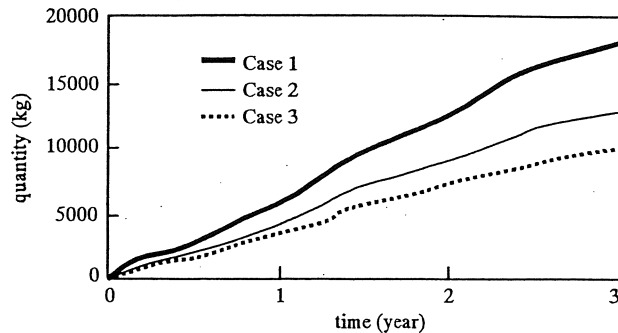


Fig. 3 Cumulative quantity of bauxite used for producing the copier drums. Case 1: the used drums are discarded. Case 2: the used drums are discarded or material-recycled. Case 3: the used drums are discarded, material-recycled, or reused.

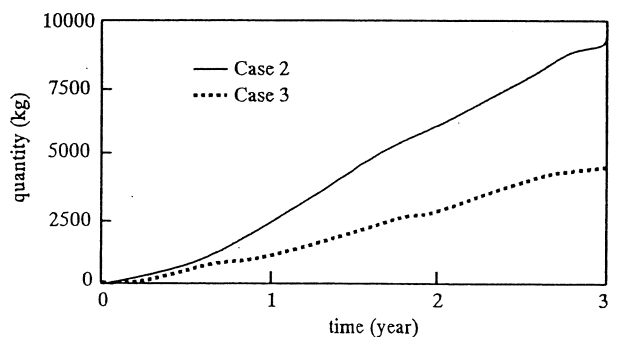


Fig. 4 Quantity of the used aluminum that the recycler has. Case 2: the used drums are discarded or material-recycled. Case 3: the used drums are discarded, material-recycled, or reused.

Figure 2 shows the life-cycle model of Case 3. However each process in the life cycle is represented as a rectangle in Fig. 2 because the complete petri-net representation of the life cycle is too complicated to be drawn. Figures 3 and 4 show the results of the simulation using the models. Figure 3 shows how material-recycling and reusing are effective in the reduction of raw material usage. However the used aluminum the recycler has is stockpiled in Case 2 and 3 as shown in Fig. 4 because the demand for the used aluminum is not enough in the case of the closed-loop recycling.

## 4. Conclusion

We proposed a method of modeling and simulating products' life cycles, and we carried out the modeling and simulation for a copier drum. Future work will extend our method to consider the evaluation of cost/benefit and environmental loads throughout the life cycle. We will also model and simulate the life cycles of more complicated products.

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