

## Probabilistic Methods in Life-Cycle Design

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**Abstract**—This paper summarizes our progress in using probabilistic methods to enable both single-domain and multiple-domain considerations in life-cycle design. As an example of a single-domain application, a probability-based reliability model that describes the effect of remanufacture on the reliability of parts and systems is outlined and experimentally verified. This reliability model is applied to fastening systems and used to calculate fastening-related remanufacture costs. These costs are integrated with other life-cycle fastening-related costs, thus enabling the simultaneous consideration of multiple domains. The combined life-cycle cost is optimized using genetic algorithms. The implementation and general results of genetic algorithm-based optimization of a product fastening and joining plan are described.

### I. INTRODUCTION

#### A. Motivation for Approach

Considerable progress has been made in environmentally conscious design. There are now a number of eco-indicators, life-cycle analysis tools, and design-for-environment (DFE) guidelines available. A recent paper [Alting & Legarth 95] provides a good overview of the field. However, it can still be very difficult to combine or compare eco-indicators, life-cycle analysis, or DFE tools with traditional requirements, performance analyses, or design tools.

Design problems are typically decomposed in vertical domains, as illustrated by the columns in Fig. 1. This vertical decomposition allows one to optimize discrete aspects of a design's performance, such as ease of assembly for example, but it may not address interrelationships between different domains. For example, a component that is easy to assemble may not be easy to disassemble or remanufacture [Shu & Flowers 95].

We view the addition of environmental design goals to traditional product design objectives as a trend towards system-oriented product design. Thus, in addition to developing an understanding of environmental design methods within various fields, there is a need for tools that designers may use to integrate results from these different methods. It will be extremely difficult to design products that best satisfy a wide range of life-cycle goals without an integrated analytical system-design method. We envision this system-oriented approach as a horizontal decomposition that spans across the vertical domains, as shown in Fig. 1. Complexity

is managed through approximation, as suggested by the layers in the figure. Each successive layer of the system model becomes more complex but also provides better estimates of a design's life-cycle performance. Initial work towards this approach is described in [Wallace et al. 95, Senin et al. 96].

In this paper, we outline a probabilistic reliability model to describe the effect of remanufacture on the reliability of parts and systems. The reliability-based life-cycle cost model applied to fastening systems links fastener design and replacement policy decisions subject to manufacture and assembly, remanufacture, maintenance and scrap-material recycling considerations. This paper will demonstrate how probabilistic modeling can be used to predict life-cycle performance in the future. We close by describing the implementation and general results of a genetic algorithm-based optimization of life-cycle fastening costs.

#### B. Background on Design for Remanufacture

For appropriate products, remanufacture is a sensible end-of-life option because it recycles products at the component level instead of the material level. This preserves the value added to parts during manufacture while avoiding the use of resources for possibly unnecessary material reprocessing. The production-batch nature of remanufacture makes possible a labor rate significantly lower than that associated with individual repair [Lund 83].

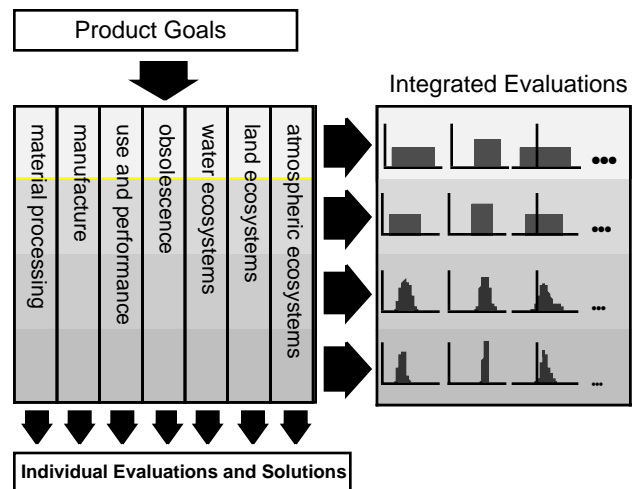


Fig. 1. Illustration of how a horizontal decomposition captures the interactions between vertically isolated domains.

While product design that facilitates any of the steps of remanufacture, including disassembly, sorting, cleaning, inspection, refurbishment and reassembly, will facilitate remanufacture, the essential goal in remanufacture is part reuse. If parts cannot be reused as is, or after refurbishment, the ease of disassembly or cleaning will not matter. To facilitate reuse, parts could either be made to never fail, or such that features prone to wear and failure are easily separable from the remainder of the part. The latter is often more practical than the former, but can contradict design-for-assembly and design-for-recycling guidelines. It is this conflict that makes it necessary to view remanufacture in the context of other life-cycle stages, in a manner we have described as horizontally integrated, instead of viewing remanufacture in isolation, or vertically. In previous work [Shu & Flowers 95], we chose to simultaneously consider the perspectives of manufacture and assembly, remanufacture, maintenance and scrap-material recycling. Because fastening and joining issues were common to all the perspectives, the costs determined by joint design for each life-cycle stage were selected for study. For manufacture and assembly, the portion of the life-cycle cost determined by the joint design is implementation and first assembly. The concern for recycling is the cost of separating materials that are not recycling compatible. For remanufacture, we are interested in how many times the joint can be disassembled and reassembled before failure, and the consequences of that failure in addition to the cost of disassembly and reassembly. It is therefore necessary to model the failure of parts that are remanufactured.

## II. PROBABILISTIC RELIABILITY MODELING

This section provides background in the vertical domain of reliability modeling. In the next section, this domain is linked with other models to create a system-based life-cycle model.

We outline a reliability model developed and detailed in [Shu 96] which describes a population of systems that undergo refurbishment activities performed during remanufacture or maintenance. We apply this model that was developed for general systems to a fastening system here. In a fastening system, the parts include the fastening elements, such as screws or rivets, as well as the parts to be fastened.

First, we present the Weibull distribution as the probability density function used to model the length of component life. Next, we present the basic behavior of the reliability model, which simulates a population of parts replaced by new parts of the same type upon failure. We then outline an experiment that verifies the basic model behavior and also demonstrates that the Weibull distribution is suitable for describing the failure characteristics for certain joints. Finally, we present the model behavior for a population of parts replaced by parts of a different type upon failure.

### A. Weibull Distribution

The Weibull distribution, appropriate for many engineering applications, is a probability density function of the length of component life described by:

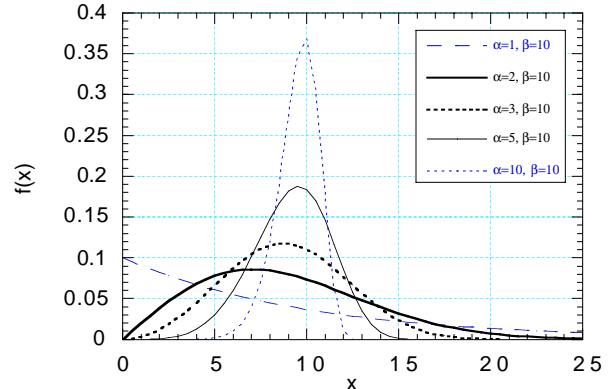


Fig. 2. Weibull densities with  $\beta=10$ .

$$f(x) = \frac{\alpha x^{\alpha-1}}{\beta^\alpha} \exp\left[-\left(\frac{x}{\beta}\right)^\alpha\right], \quad 0 \leq x < \infty$$

where  $\alpha$  and  $\beta$  are the parameters of the distribution.

Fig. 2 plots Weibull distributions with a  $\beta$  of 10 paired with  $\alpha$  equal to 1, 2, 3, 5 and 10. Alpha is the shape parameter; the higher the value of  $\alpha$ , the less spread there is about the expected value of time to failure. Beta is the scale parameter; as  $\alpha$  increases, the peak of the distribution approaches  $\beta$ . An  $\alpha$  of 1 yields the negative exponential distribution.

Given a population of parts or systems with a particular distribution, the probability density function can also be used to predict the proportion of the population that fails as a function of time.

### B. Reliability Model Description

The reliability model simulates the replacement of a population of systems upon failure with new parts. The model represents the population of systems as a collection of populations of the constituent parts. The parts have independent and possibly different distributions of time to failure. Thus, the population corresponding to each part has an associated time-to-failure distribution, and parts are treated as members of their respective populations. During the simulation, the part ages of each population are tracked with time to determine the failure characteristics of the corresponding part. Failure of the part results in replacement by a new part of the same or different type, or in replacement of the entire system with a same or different system.

The possibility of system modification is not included in many reliability models, where replacement is limited to a component of the same type. This additional capability is motivated by common practices in remanufacture. Many refurbishment processes change the reliability characteristics by altering the system configuration. For example, screws are often replaced with larger, usually coarser-thread screws when the boss is stripped by the original screw. Inserts are also installed during remanufacture, which increases the number of disassembly and reassembly cycles the part can survive. In the remanufacture of toner cartridges that were not designed for disassembly, heat stakes are drilled out during disassembly, and threaded fasteners are used for reassembly.

Brads are used in place of screws, and vice versa, in the remanufacture of starter-solenoid assemblies.

In this model, the choice of repair policy determines actions executed upon part failure. In practice, corporate refurbishment policy significantly affects both the system reliability and the consequent remanufacture cost of a given original design. Some companies may choose to replace all parts of a particular type without inspection, either due to product reconfiguration or past reliability problems, while others will replace based on either actual part failure or projected life remaining.

Currently, this model considers series systems where the density of time to failure of each component is represented by the two-parameter Weibull distribution. The extension of this methodology to use other distributions is fairly straightforward and a matter of implementation.

### C. Simulation of Replacement upon Failure with Same Parts

Fig. 3 plots the average age of constant-size populations of identical parts that are replaced by new components of the same type upon failure. Each curve represents a population of parts with a particular Weibull distribution of time to failure. The plots shown correspond to the Weibull distributions with arbitrarily selected parameters of Fig. 2.

Several characteristics of Fig. 3 are of interest. First, the average age eventually reaches a steady state value. The value of the steady state age depends upon Weibull parameters  $\alpha$  and  $\beta$ . The dependence on  $\beta$  is not surprising; higher values of  $\beta$  for a given set of  $\alpha$ 's yield higher values for expected time to failure and thus average age. Alpha affects both the steady state value and the degree of oscillation. As  $\alpha$  increases, the time span during which a majority of parts fail decreases. For high values of  $\alpha$ , very few parts will fail until time  $\beta$ , after which almost all the parts will fail immediately. During the low-failure period, the average age will increase monotonically. Then as increasingly large numbers of parts fail, the replacement of a significant portion of the population causes the average age to drop until the wave of failure is over. The newly installed base of parts then ages steadily until the next failure wave. During each oscillation, a number of parts fail outside the time window during which most of the population fail. The population thus becomes more age-diversified, and the oscillations in average age die down. The higher the value of  $\alpha$ , the fewer parts fail outside the tighter expected failure period, and thus the greater the oscillations in average age and the longer it takes for diffusion to occur. As  $\alpha$  increases, the mean of the average age approaches  $\beta/2$ . This is intuitive when one considers the upper bound as  $\alpha$  approaches infinity. Physically, such a distribution of time to failure implies that no parts fail until time  $=\beta$ , at which time all the parts fail. Therefore, this population would have a saw-toothed average age plot that does not decay and is bounded between 0 and  $\beta$ .

Fig. 4 plots the replacement parts cost corresponding to Fig. 3. For ease of comparison, all parts were assigned an identical normalized cost proportional to  $\beta$ . In reality, part cost is likely to be a function of both  $\alpha$  and  $\beta$ . The trends of

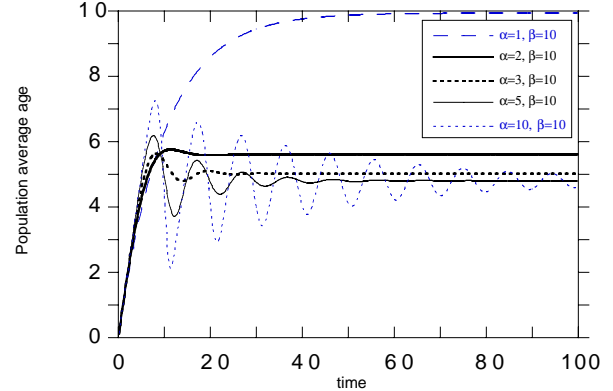


Fig. 3. Basic simulation population average age.

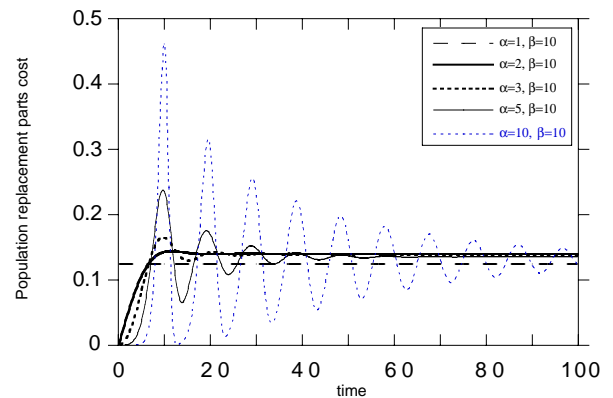


Fig. 4. Basic simulation replacement parts cost.

Fig. 4 are consistent with those of Fig. 3. The replacement part cost increases as average age decreases since parts are being replaced at a higher rate. Steady state replacement costs are higher for lower steady state average ages.

### D. Experimental Verification of Reliability Model

An experiment was performed to verify the basic behavior of the model. This experiment applied the model to a fastening system and involved obtaining data on the number of disassembly and reassembly cycles before a screw strips a hole in plastic.

Wear and failure of mechanical elements often occur due to relative motion between parts. The failure characteristics of a part are determined by not only that part's material and geometry, but also by interactions with neighboring parts. For example, for a given set of operating conditions, the failure characteristics of a gear depend heavily on the gear with which it meshes. Similarly for joints, the failure characteristics of a boss depend on the geometry and material of both the boss and the screw. Relative motion between elements of a joint may occur during product operation, but certainly during disassembly and reassembly. The disassembly and reassembly method thus also contributes to the failure characteristics of the joint. Focusing on the failure that occurs during disassembly and reassembly, the number of

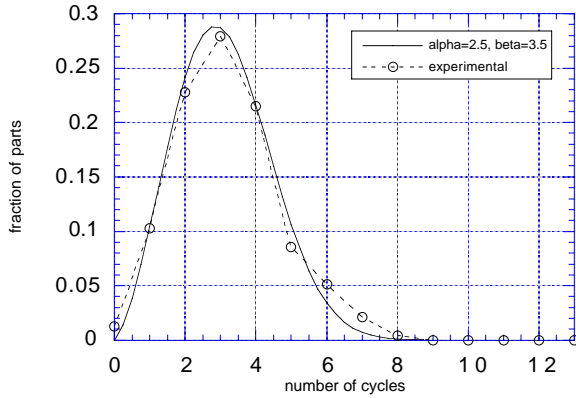


Fig. 5. Sample cycles-to-failure histogram vs. Weibull distribution of  $\alpha=2.5$ ,  $\beta=3.5$ .

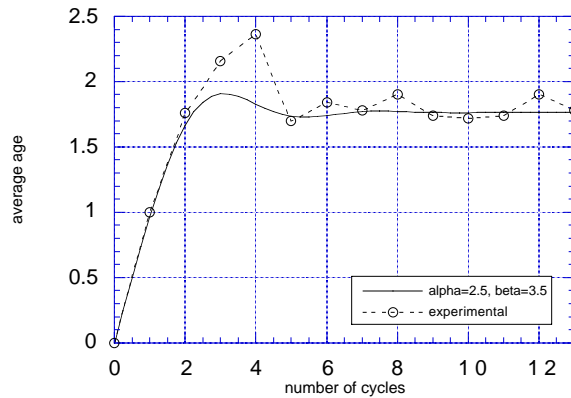


Fig. 6. Average age obtained experimentally vs. through simulation using Weibull parameters  $\alpha=2.5$ ,  $\beta=3.5$ .

disassembly and reassembly cycles represent the time scale used to describe the reliabilities of joint elements.

For the experiment, a grid of holes was drilled in a sheet of polypropylene. Thread-forming screws were inserted and removed using a power screwdriver at a constant torque until the screw continued to spin when fully inserted.

The number of rows of holes represents the number of systems in the sample. A sample size of 50 was used. When a hole fails, ‘part replacement’ involves using the next hole in the same row. A screw removal-and-insertion cycle performed on the sample constitutes a time step. The number of screw removal-and-insertion cycles until failure was recorded for each hole. This was used to obtain a distribution of number of cycles to failure for the sample. The number of cycles survived by each active hole averaged over the sample at each time step yields the average age plot. The data associated with the final holes were not used to obtain the cycles-to-failure distribution because those holes had not failed yet, but they were used to calculate the average age.

Fig. 5 compares the sample histogram of cycles to hole failure with the Weibull distribution that produced the least-squared error between the experimental data points and the

values determined by the distribution. Fig. 6 compares the average age yielded experimentally with that produced through simulation using the above Weibull distribution. The agreement is reasonable considering the relatively small sample size of 50. The smaller sample size is much more sensitive to outliers and thus displays a greater noise level than if a larger sample size were used.

### E. Simulation of Replacement with Different Parts

We now present the simulation results for a population of parts that are replaced upon failure by parts with different failure parameters. Subsequent failure of replacement parts result in replacement by the same parts, i.e., parts of the original type are not reintroduced into the population.

Figures 7 and 8 chart the replacement of an initial population of parts with arbitrarily selected Weibull parameters  $\alpha=3$ ,  $\beta=10$ , hereon denoted (3,10), with parts of Weibull parameters  $\alpha=10$ ,  $\beta=10$  (10,10). Subsequent replacement of failed (10,10) parts are with the same (10,10) parts. For reference, replacement of an initial population of (3,10) parts by the same (3,10) parts and replacement of an initial population of (10,10) parts by the same (10,10) parts are also plotted.

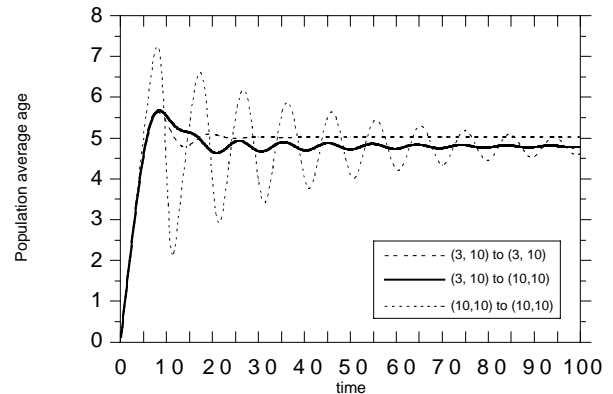


Fig. 7. Average age of  $\alpha=3$ ,  $\beta=10$  parts replaced by  $\alpha=10$ ,  $\beta=10$  parts.

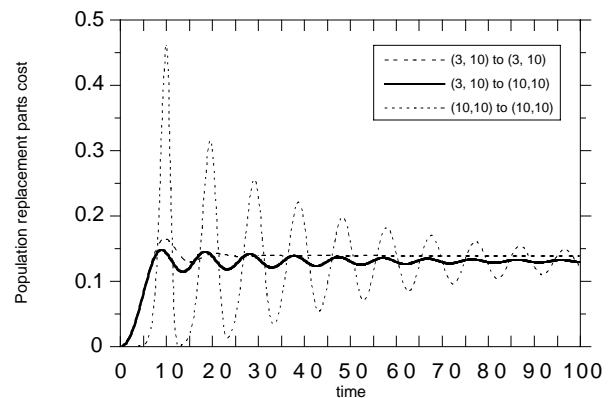


Fig. 8. Replacement parts cost of  $\alpha=3$ ,  $\beta=10$  population replaced by  $\alpha=10$ ,  $\beta=10$  parts.



Of interest in Figures 7 and 8 are the phase shift and reduced oscillation from the (10,10)-to-(10,10) curve to the (3,10)-to-(10,10) curve. An original population of (3,10) parts fail earlier and with more spread between time of failures than an original population of (10,10) parts. Therefore, the first replacement batch of (10,10) parts appear earlier and more staggered over time than the first replacement batch for the population that began with (10,10) parts. The effect of this initial difference carries over to subsequent oscillations.

### III. PROBABILITY-BASED OPTIMIZATION OF LIFE-CYCLE FASTENING AND JOINING COST

The previous sections illustrated that the remanufacture costs of a product are affected by the original part design, actions performed during remanufacture, and the repair policy that determines the consequence of part failure. The combination of these factors determines the remanufacture cost, possibly totaled over several remanufacture cycles. In this section, the remanufacture cost is combined with costs from other life-cycle stages to create a horizontally integrated system cost model. The combined cost is optimized using genetic algorithms, a probabilistic search method inspired by biological evolution used for combinatorial optimization.

We first present a brief overview of genetic algorithms, details on which can be found in [Goldberg 89]. Next, we outline the implementation of a product fastening plan optimization. We close by summarizing general trends of the optimization results on a sample search space described in [Shu & Flowers 96].

#### A. Genetic Algorithm Fundamentals

Genetic algorithms are used to simulate the evolution of design solutions. A design solution is represented as a single chromosome. Multiple solutions exist as a population of chromosomes that is evolved toward superior solutions. Superiority of a solution is determined by an objective function that represents a quantity to be minimized or maximized.

An initial population of solutions is created upon startup of the genetic algorithm. Evolution is executed through a process of selection, crossover and mutation of members of the population. First, chromosomes are selected based on fitness to be parents for the following generation. Fitness is a scaled value of the chromosome's objective function value. Crossover of two parent chromosomes involves combining parts of the parents to yield chromosomes representing new design solutions. Mutation involves a random alteration of part of a particular chromosome and is performed to maintain diversity in the population. The original population is replaced in part or whole by new chromosomes yielded by these operations. This process continues until either a number of generations or some convergence criterion on the objective function has been achieved.

#### B. Genetic Algorithm Optimization of Fastening Plan

The genetic algorithm library developed by [Wall 96] was used for our implementation. For our application, the chromosome, or possible solution, represents a fastening or

joining plan. The plan consists of the initial fastening method, the subsequent disassembly (and reassembly) method, and the repair policy. The objective function to be minimized is the life-cycle assembly, disassembly and reassembly cost as determined by the fastening plan.

During initialization of the chromosome, a general class of fastening methods, e.g., integral fasteners, threaded fasteners etc., is randomly selected. Based on this class, each part of the chromosome is selected from a predefined, appropriate set of alleles (possible values for parts of the chromosome) as illustrated in Fig. 9. During crossover, the types of methods represented by the parents are checked for compatibility before they are crossed over. This is to prevent nonsensical solutions, an instance of which would combine a snap fit fastening method with an unscrew disassembly method. Both the number and location of crossover points are randomly generated. Mutation, as illustrated in Fig. 11, involves the random selection of one of the three parts of the chromosome and reselecting a value from the appropriate set of alleles.

The objective function calculates the life-cycle fastening cost of the population of joints as follows. The cost associated with manufacture and assembly includes the cost of fastening or joining method implementation, such as drilling holes to accommodate screws, the cost of the fastening elements, and the assembly cost, such as the installation of screws.

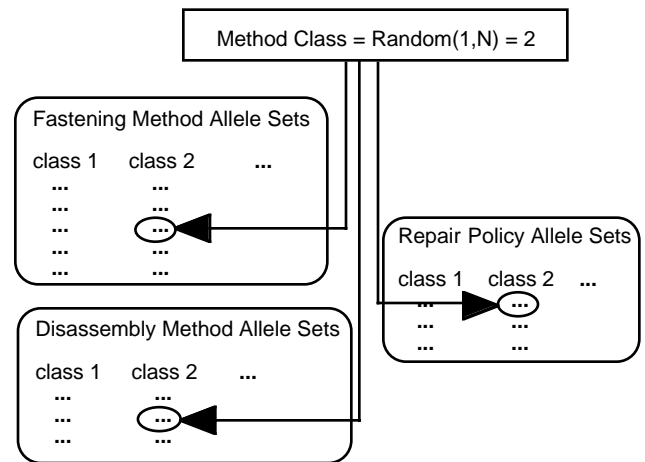


Fig. 9. Fastening-plan chromosome initialization.

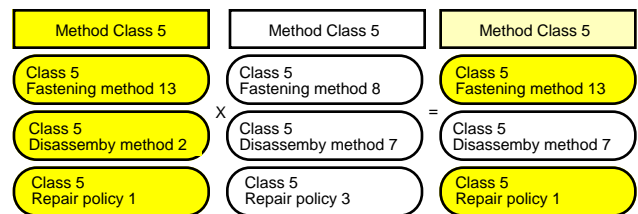


Fig. 10. Two-point, single-child crossover.

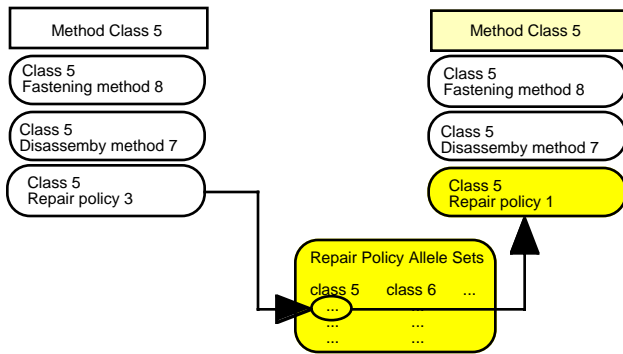


Fig. 11. Single-point mutation

The cost for recycling that is determined by the fastening or joining method is the cost of separating materials that are not recycling compatible with each other, such as the removal of metal screws from plastic parts. The number of remanufacture or disassembly and reassembly cycles is selected by the designer for the population of systems. During each cycle, the age distributions of the populations of constituent parts are used to calculate the rates of failure. The portion of the population that fails is replaced by new parts. The age of this portion is reset to zero. The age of the surviving portion of the population is incremented. The cost of each remanufacture cycle includes the cost of disassembly and reassembly of the population using the appropriate disassembly and reassembly methods, and the cost of either part replacement or reconfiguration for the portion of the population that fails.

The general trends of the optimization results on a sample search space from [Shu & Flowers 96] are as follows. Reconfiguring a failed system can be cheaper than replacement of the failed part. Examples of reconfiguration include using a larger, coarser-thread screw in a boss stripped by the original screw. Also, if a failed snap fit is in an inconspicuous location, a hole can be drilled to accommodate a screw. It is also cheaper to use a disassembly method that may be slower, but is less likely to cause damage to a part that is difficult to reconfigure or costly to replace.

#### IV. SUMMARY AND FUTURE WORK

In this paper we have described how problems are often vertically decomposed into individual disciplines or specialties. This type of in-depth approach is essential to the development of expertise, but what we describe as a horizontally integrated system model is also needed to address the interactions between different phases of the product life-cycle.

First, a probabilistic model applied to describe the reliability of fasteners was presented. Then, this model was combined with other life-cycle cost models so that manufacture and assembly, remanufacture, maintenance and scrap-material recycling costs could be considered simultaneously.

The reliability-based remanufacturing life-cycle cost model was linked to a genetic algorithm-based search methodology to optimize both joint design for remanufacturability and replacement policy decisions subject to life-cycle cost goals. The trends of the optimization results suggest the use of disassembly methods that are slower, but less likely to cause damage during remanufacture of expensive parts. Facilitating the reconfiguration of parts, through the use of pre-molded holes next to failure-prone snaps for instance, would also help enable part reuse.

The reliability model will be further developed to include both series and parallel systems, where component failure rates can be represented by a variety of distributions. A larger database will be also developed to increase the search space for the optimization of life-cycle costs.

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