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RELIABILITY ANALYSIS OF NON-CONSTANT-SIZE PART POPULATIONS IN DESIGN FOR REMANUFACTURE

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ABSTRACT

Remanufacture offers significant economic and ecological advantages over other end-of-life options. The goal of this research is to enable estimation of cost due to replacement of parts in systems undergoing remanufacture.

In our previous work, a novel repairable-system reliability model that allows system modifications was developed to describe a population of systems undergoing remanufacture. In this paper, the reliability model is modified to accommodate population size changes during the replacement process. The effects of two types of disturbances to population size, pulse and continuous, on the replacement rate behavior are studied. Analysis of actual industrial data is presented as an example of population replacement under continuous disturbance. This example confirms that a simulation using the reliability model proposed in this paper yields an estimation for replacement rate with acceptable error bound.

Keywords: Environmentally conscious design, remanufacture, reliability, repairable systems, life-cycle replacement cost.

INTRODUCTION

Life-cycle design considers product life from conception to disposal, with the goal of achieving more resource-efficient designs. There has been growing interest in life-cycle design over the past decade (Finger and Dixon, 1989; Kalvan-Seshu and Bras, 1998). Options to disposal at product end-of-life include scrap-material recycling, remanufacture and repair/reuse.

Remanufacture offers significant economic and ecological advantages over other end-of-life options for appropriate products (Parker, 1997). Remanufacture consists of the production-batch disassembly, cleaning, replacement and refurbishment of worn parts in defective or obsolete products. It restores worn and discarded durable products to a like-new condition (Lund, 1996). In this context, this paper is concerned with the development of mathematical tools for the economic analysis of systems undergoing remanufacturing. Such tools can help manufacturers plan an economical and environmentally sensible option to product landfill or incineration at end-of-life.

Remanufacture can be considered as recycling at the component level instead of the material level. Therefore, it avoids possibly unnecessary resource consumption of scrap-material recycling while preserving the value added to a component during manufacture (Lund, 1983). The production-batch and off-site nature of remanufacture results in a labor cost significantly lower than that required for individual repair. Remanufacture is estimated to save between 40 and 60 percent of the cost of manufacturing a completely new product, while requiring only 20 percent of the energy, (Hormozi et al., 1996).

There exists a need for integrating environmental, including remanufacturability, considerations with traditional design assessment in order to make better tradeoffs between complex sets of design objectives, such as performance and economic factors (Jackson and Wallace, 1997). Life-cycle cost, the sum of all costs incurred during the lifetime of a product, is an important criterion for decision making in design (Dhillon, 1989). Thus, the evaluation of the portion of life-cycle cost due to replacement of parts in systems undergoing remanufacture is the objective of this paper.

In previous work (Jiang et al., 1998), the replacement cost of a population of parts was related to reliability indices such as part replacement rate. The stochastic behavior of a reliability model that is suitable for describing systems undergoing remanufacture was investigated. It was proven that the replacement process will reach steady state and become a Superimposed Renewal Process. The expressions for the reliability indices at steady state were obtained to facilitate life-cycle cost estimation.

The reliability model previously developed assumed a constant population size. In practice, however, population sizes may vary as a function of time. Some systems may not be replaced after failure, causing a decrease in population size, while other systems may be added after the initial population

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was put into service, causing an increase in population size. Such system populations are referred herein as non-constant-size system populations. A system is comprised of parts. Thus, a population of systems is modeled as a collection of populations of parts. In this paper, the replacement rate behavior of a non-constant-size part population is studied.

NOMENCLATURE

The terms used in the reliability modeling and statistical analysis of repairable systems are:

Terms defined for parts

- x — age of part, measured in time, a real variable
- $f(x)$ — failure density function of parts

$$\int_0^\infty f(x)dx = 1$$

- $i(x)$ — failure probability function of parts

$$i(x) = \int_0^x f(x)dx$$

- $f_1(x)$ — failure density function of original parts in part population
- $f_2(x)$ — failure density function of replacement parts
- p — replacement cost of one part
- α — shape parameter of Weibull distribution
- β — scale parameter of Weibull distribution

Terms defined for part populations

- m — number of sockets in a series system
- M — number of parts in part population
- t — time elapsed from when population of parts was first put into service
- Δt — interarrival time between scheduled replacements
- $v(t)$ — at time t, replacement rate of part population, i.e., average fraction of parts in population being replaced per unit time
- t_n — denotation for time measured in units of Δt, where n is a non-negative integer defined as $n=t_n/\Delta t$
- $q_i(t_n)$ — age distributions, i.e., at time t_n, fraction of parts with age $i\Delta t$, $i=0,1,\ldots n$
- $N(t)$ — number of failures that occur during $(0,t]$
- $\{f_1(x)\leftarrow f_2(x)\}$ — replacement process of a constant-size part population, where original parts have a failure density function $f_1(x)$ and replacement parts have a failure density function $f_2(x)$
- $c(t)$ — at time t, replacement cost per unit time of part population, a function of p, $v(t)$, M
- $C(t)$ — at time t, cumulative replacement cost function of part population

RELIABILITY MODEL FOR SYSTEMS UNDERGOING REMANUFACTURE

This section summarizes a reliability model for systems undergoing remanufacture developed in previous work (Shu and Flowers, 1998; Jiang et al., 1998).

The reliability model for systems undergoing remanufacture describes a population of m-socket series systems. The population size is constant. The population of M systems is represented as a collection of m populations of constituent parts, and parts are treated as members of their respective populations.

In remanufacture, part failure results in replacement of the part with a component of the same or different type. The remaining system either remains unchanged or is reconfigured to accommodate the replacement part. The failure density function of the original parts in the population is denoted $f_1(x)$. The failure density function of the replacement parts is denoted $f_2(x)$. Thus, the replacement process of a constant-size part population is denoted $\{f_1(x)\leftarrow f_2(x)\}$.

If $f_1(x)$ and $f_2(x)$ are the same, i.e., $f_1(x)=f_2(x)=f(x)$, then system modification does not occur through the replacement process. However, if $f_2(x)$ and $f_1(x)$ differ, then system modification does occur during the replacement process.

The age distributions of each of the part populations are tracked to determine the reliability of the composite system population. The fractions of parts in the population, whose ages equal $i\Delta t$, where $i=0,1,\ldots n$, at time $t_n=n\Delta t$, are:

$$
\begin{aligned}
q_n(t_n) &= q_0(t_0)\left[1-\int_0^{n\Delta t} f_1(x)dx\right] \\
q_{n-1}(t_n) &= q_0(t_1)\left[1-\int_0^{(n-1)\Delta t} f_2(x)dx\right] \\
&\vdots \\
q_2(t_n) &= q_0(t_{n-2})\left[1-\int_0^{2\Delta t} f_2(x)dx\right] \\
q_1(t_n) &= q_0(t_{n-1})\left[1-\int_0^{\Delta t} f_2(x)dx\right] \\
q_0(t_n) &= q_0(t_0)\left[\int_{(n-1)\Delta t}^{n\Delta t} f_1(x)dx\right] \\
&\quad +q_0(t_1)\left[\int_{(n-2)\Delta t}^{(n-1)\Delta t} f_2(x)dx\right]+\ldots \\
&\quad +q_0(t_{n-2})\left[\int_{\Delta t}^{2\Delta t} f_2(x)dx\right] \\
&\quad +q_0(t_{n-1})\left[\int_0^{\Delta t} f_2(x)dx\right]. \qquad (1)
\end{aligned}
$$

After obtaining the value of $q_o(t_n)$, the fraction of parts failed and replaced at time t_n, the replacement rate at time $t_n=n\Delta t$, i.e., $v(n\Delta t)$, is calculated as:

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$$v(t_n) = \frac{q_0(t_n)}{\Delta t}. \quad (2)$$

The objective of this reliability modeling is to evaluate life-cycle replacement cost. Thus, the replacement cost for the part population per unit time at time $t=t_n$, is related to the part population replacement rate and is modeled as:

$$c(t) = pMv(t), \quad (3)$$

where p is the replacement cost of one part and M is the number of parts in the part population. It is noted that the part population replacement rate is of fundamental importance in determining replacement cost.

The population cumulative replacement cost of the part population from time 0 to time t is

$$C(t) = \int_0^t c(t)dt. \quad (4)$$

Jiang et al. (1998) investigated the stochastic behavior of the reliability model to facilitate estimation of life-cycle cost. It was proven theoretically that the replacement process with system modifications under a perfectly maintained repair policy reaches steady state, and is a Superimposed Renewal Process (SRP). At the limit, as $t \rightarrow \infty$, a SRP behaves like a Homogeneous Poisson Process (HPP) with a constant hazard rate, which can be represented by an exponential distribution (Drenick, 1960). In a perfectly maintained process with system modifications, the steady state value for a replacement rate depends only on the failure density function of replacement parts. Specifically, it converges to the reciprocal of the mean life of replacement parts.

Representation of Failure Density Function

In the reliability model represented in Equation (1), there exists no restriction on the failure density function ***f(x)***. In this paper, the two-parameter Weibull distribution is used to represent the failure density function ***f(x)*** for simulation purposes. Extension to other distributions is fairly straightforward.

The two-parameter Weibull distribution is:

$$f(x) = \begin{cases} \dfrac{\alpha x^{\alpha-1}}{\beta^\alpha} \exp\left[-\left(\dfrac{x}{\beta}\right)^\alpha\right], & 0 \le x \le \infty \\ 0, & elsewhere, \end{cases} \quad (5)$$

where $\alpha>0$, $\beta>0$, and x is the age of the part.

The Weibull failure probability function is:

$$i(x) = \int_0^x f(x)dx$$
$$= -\exp\left(-\left(\frac{x}{\beta}\right)^\alpha\right)\Bigg|_0^x$$
$$= 1 - \exp\left(-\left(\frac{x}{\beta}\right)^\alpha\right). \quad (6)$$

RELIABILITY MODEL FOR NON-CONSTANT-SIZE POPULATIONS

In this section, the reliability model is modified to accommodate population size changes during the replacement process. The variation of a population's size can be treated as a random disturbance. Two types of disturbances are considered:

1. Pulse disturbance, where the variation only happens once during the time under consideration, Figure 1a; and,
2. Continuous disturbance, where population size variations occur at random time intervals and at random amplitudes, Figure 1b.

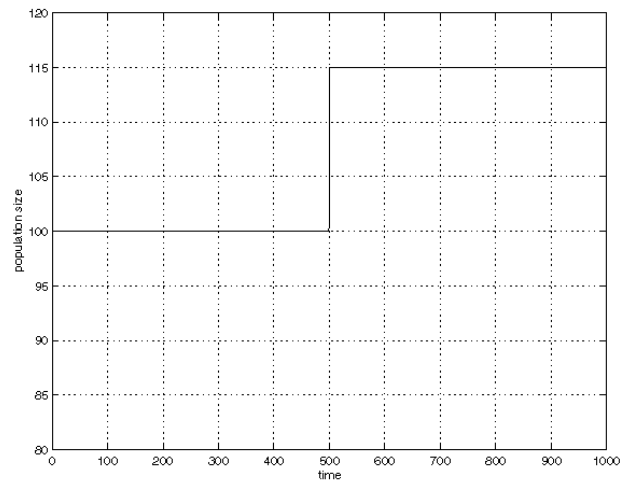

Figure 1a: Pulse disturbance to population size.

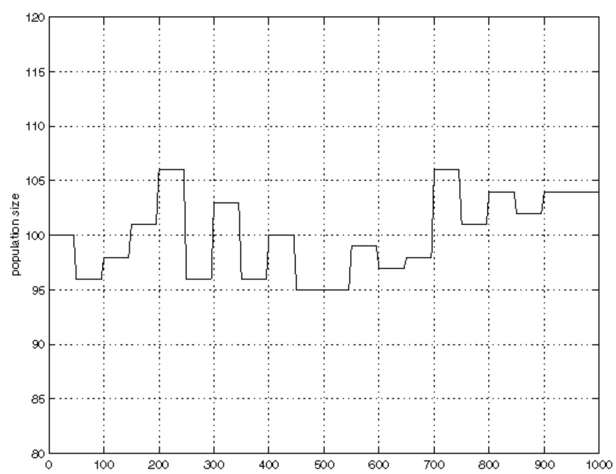

Figure 1b: Continuous disturbance to population size.

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Replacement Rate for Size Increases

For a replacement process where the population size increases, the total population, (i.e., the part population after the size increase), can be viewed as a combination of two separate populations. The first is the original part population, installed at time t_0, whose fraction in the total population after the size increase is r_1. The second population, added at time t_m, has fraction in the total population r_2, where $r_1+r_2=1$.

The replacement process for the original part population starts at time t_0, while the replacement process for the newly added part population starts at time t_m. After new parts have been added, the replacement rate behavior for the total population is a combination of the two replacement processes.

Let us suppose that a time period of $n\Delta t$ has passed after the new parts have been added. The replacement rate at time $t_{m+n}=(m+n)\Delta t$ is to be calculated. In the total population, the fraction of parts which failed during time $t_{m+n}=(m+n)\Delta t$ to $t_{m+n+1}=(m+n+1)\Delta t$ can be expressed as:

$$q_0(t_{m+n}) = r_1 q_0{}^1(t_{m+n}) + r_2 q_0{}^2(t_n) \tag{7}$$

where $q_0^1\ (t_{m+n})$ is the fraction of parts that failed of the original part population and $q_0^2(t_n)$ is the fraction of parts that failed of the newly added part population. $q_0^1\ (t_{m+n})$ and $q_0^2(t_n)$ are calculated using Equation (1). According to Equation (2), at time t_{m+n}, the replacement rate for the entire population is

$$\begin{aligned} v((m+n)\Delta t) &= \frac{q_0(t_{m+n})}{\Delta t} \\ &= \frac{r_1 q_0{}^1(t_{m+n}) + r_2 q_0{}^2(t_n)}{\Delta t} \end{aligned} \tag{8}$$

Replacement Rate for Size Decreases

During the part population replacement process, the population size may also decrease due to the removal of a fraction of parts for various reasons. For example, some systems may be discontinued and not replaced. If the replacement rate for above case is considered, the age groups to which the removed parts belong must be specified. Otherwise, the information for the part population age distribution cannot be determined.

Herein, only one condition for size decrease is considered, i.e., some of the parts that failed are not replaced. Those unreplaced failed parts can be viewed as being removed from the original population. Thus, the population size is decreased by a fraction. Naturally, the fraction of size decrease cannot be greater than the fraction of parts that failed at that time.

For a part population whose size decreases, the original part population can be viewed as a combination of two separate part populations. The first one is the population of parts remaining in the process after the size decrease, whose fraction in the original part population is s_1. The second one is the part population that failed at time t_m, and is not replaced, whose fraction in the original population is s_2, where $s_1+s_2=1$. Note that, s_2 cannot be greater than the fraction of parts failed at t_m.

For the sake of discussion, let us suppose that the removed parts were actually replaced, namely, added back into the process as a new population. Then, they would be experiencing a replacement process that starts at time t_m, whereas the replacement process for the original part population had started at time t_0. Similar to the replacement process when the population size increases, the remaining part population and the removed (but instantly re-added) part population will experience different replacement processes.

Suppose a period of $n\Delta t$ has passed after the new parts are "pseudo-added", then, the replacement rate at time $t_{m+n}=(m+n)\Delta t$ needs to be calculated. For the remaining population, the fraction of parts that failed during time $t_{m+n}=(m+n)\Delta t$ to $t_{m+n+1}=(m+n+1)\Delta t$ can be expressed as

$$q_0(t_{m+n}) = \frac{\left(q_0^1(t_{m+n}) - s_2 q_0^2(t_n)\right)}{s_1}, \tag{9}$$

where $q_0^1\ (t_{m+n})$ is the fraction of parts that failed in the original part population and $q_0^2(t_n)$ is the fraction of parts that failed in the pseudo-added part population. Then, at time t_{m+n}, the replacement rate of the remaining population is

$$\begin{aligned} v((m+n)\Delta t) &= \frac{q_0(t_{m+n})}{\Delta t} \\ &= \frac{\left(q_0^1(t_{m+n}) - s_2 q_0^2(t_n)\right)}{s_1 \Delta t}, \end{aligned} \tag{10}$$

where $q_0^1\ (t_{m+n})$, $q_0^2(t_n)$ are calculated using Equation (1).

RELIABILITY ANALYSIS FOR PULSE DISTURBANCE

In this section, only a replacement process under pulse disturbance, causing size increase, will be discussed. The expansion to pulse disturbance causing size decrease is straightforward and detailed in (Jiang, 1999).

Analysis of Replacement Process

When a new sub-population of systems are added to a population, the configuration of the new systems may be the same as that of the original population, or it may be different. Thus, for a part population under consideration, the failure density function of those newly added parts may be the same as that of the initial part population, or different. Assuming that when a different type of parts replaces failed parts, the original type of parts will not be reintroduced into the population, three situations may exist:

1. The process begins with $\{f(x)\leftarrow f(x)\}$. At time t_m, $m>0$, new parts with a failure density function $f(x)$ are added into the original part population (i.e., pulse disturbance). After time t_m, failed parts continue to be replaced with new parts with the same failure density function, $f(x)$. This process, denoted $\{\{f(x)\leftarrow f(x)\},\ +f(x),\ f(x)\}$, consists of three components, the original replacement process, $\{f(x)\leftarrow f(x)\}$, the addition of new parts, $+f(x)$, and the failure density function of the replacement parts after size increase, $f(x)$.
2. The process begins with $\{f_1(x)\leftarrow f_1(x)\}$. At time t_m, $m>0$, new parts with a failure density function $f_1(x)$ are added into the original part population. After time t_m, failed parts are replaced with new parts with a different density function, $f_2(x)$. This process is denoted $\{\{f_1(x)\leftarrow f_1(x)\},\ +\ f_1(x),\ f_2(x)\}$. Note that, in this process, the replacement policy changes from replacement-with-same to replacement-with-different type of parts, after the new sub-population of parts is added.
3. The process begins with $\{f_1(x)\leftarrow f_2(x)\}$. At time t_m, $m>0$, new parts with a failure density function $f_2(x)$ are added into the original part population. After time t_m, failed parts are replaced with new parts with a failure density function $f_2(x)$. This process is denoted $\{\{f_1(x)\leftarrow f_2(x)\},\ +\ f_2(x),\ f_2(x)\}$.

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When the replacement policy changes after the new parts are added, as in Case 2, the reliability model must be modified as follows:

$$q_i(t_{m+n}) = q_0(t_{n+m-i})\left[1-\int_0^{i\Delta t} f_1(x)dx\right],$$

$$i=n+1, \ldots, n+m$$

$$q_i(t_{m+n}) = q_0(t_{n+m-i})\left[1-\int_0^{i\Delta t} f_2(x)dx\right],$$

$$i=1, \ldots, n$$

$$q_0(t_{m+n}) = \sum_{j=n+1}^{n+m} q_0(t_{n+m-j})\int_{(j-1)\Delta t}^{j\Delta t} f_1(x)dx + \sum_{j=1}^{n} q_0(t_{n+m-j})\int_{(j-1)\Delta t}^{j\Delta t} f_2(x)dx \qquad (11)$$

As stated in (Jiang et al., 1998), for a perfectly maintained process with $\{f_1(x) \leftarrow f_2(x)\}$, the replacement rate will reach a constant value that equals the reciprocal of the mean life of replacement parts. Thus, it can be conjectured that the replacement rate of a process subject to pulse disturbance will also reach a steady state value that depends only on the replacement parts.

The age distribution of a part population at steady state can be calculated using expressions developed in (Jiang et al., 1998). When new parts are added into this original part population via a pulse disturbance, the age distribution of the population is subjected to a transient period until the new part population reaches steady state once again.

Examples

Simulations were run to track the replacement rate behavior for a non-constant-size part population with a pulse size increase. To show the effect of a size change more clearly, time t_m is chosen as a time instance after the original population has reached steady state.

Exemplary replacement rates for the above three processes, $\{\{f(x) \leftarrow f(x)\}, + f(x), f(x)\}$, $\{\{f_1(x) \leftarrow f_1(x)\}, + f_1(x), f_2(x)\}$, and $\{\{f_1(x) \leftarrow f_2(x)\}, + f_2(x), f_2(x)\}$ are shown in Figures 2, 3 and 4, respectively, for three different replacement ratios, r_1. As previously stated, r_1 is the fraction of original population with respect to the overall population size after the addition of new parts. In Figure 2, the failure density function $f(x)$ is $(3,4)$, i.e., α=3, β=4. In Figure 3, $f_1(x)$ is $(3,4)$ and $f_2(x)$ is $(3,10)$. In Figure 4, $f_1(x)$ is $(10,10)$ and $f_2(x)$ is $(3,10)$. Time t_m is set to ***40, 60, and 60*** in Figures 2, 3, and 4, respectively.

As expected, in Figures 2 through 4, the replacement rate exhibits transient behavior when new parts are added. Since the original part population has already reached steady state, it can be concluded that the transient behavior is caused solely by the new part population added at time t_m. As r_1 increases, the disturbance decreases. The replacement rate reaches a steady state value that is determined by the failure density function of the replacement parts.

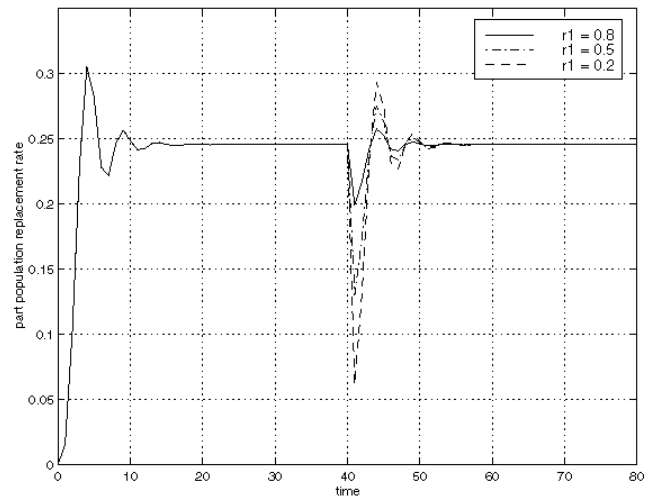

Figure 2: Replacement rate curves for $\{\{f(x) \leftarrow f(x)\}, + f(x), f(x)\}$.

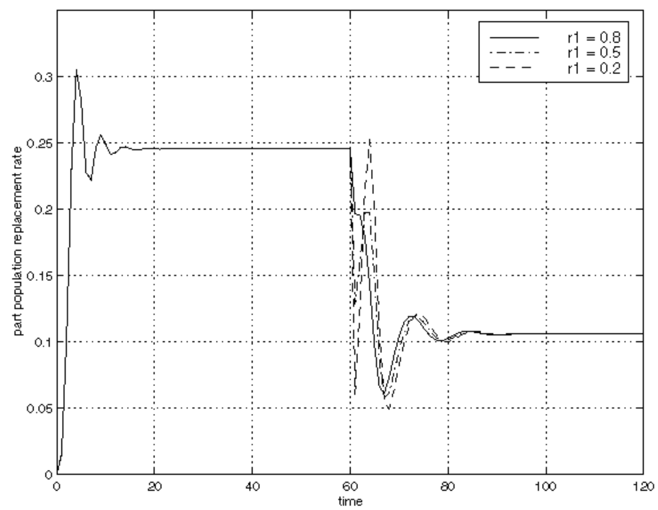

Figure 3: Replacement rate curves for $\{\{f_1(x) \leftarrow f_1(x)\}, + f_1(x), f_2(x)\}$.

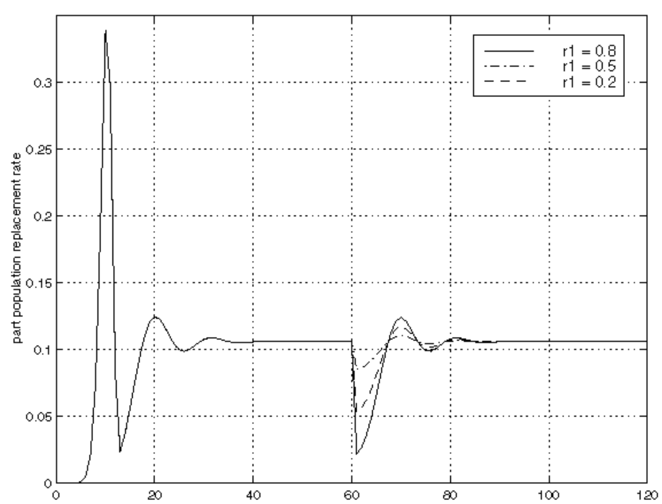

Figure 4: Replacement rate curves for $\{\{f_1(x) \leftarrow f_2(x)\}, + f_2(x), f_2(x)\}$.

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RELIABILITY ANALYSIS FOR CONTINUOUS DISTURBANCE

In this section, the replacement rate behavior of a part population under continuous disturbance is studied.

The previous section showed that when a pulse disturbance is applied to a replacement process, the replacement rate exhibits a transient behavior. It can be conjectured that the replacement rate under continuous disturbance will also fluctuate since a continuous disturbance can be viewed as a sequence of pulse disturbances.

Example

Let the original part population size be denoted by $\boldsymbol{M}$. Let the deviations from the population size $\boldsymbol{M}$ be normally distributed,

$$n(x;\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\left(\frac{1}{2}\right)\left[\left(x-\mu\right)/\sigma\right]^2},$$
$$-\infty < x < \infty, \qquad (12)$$

where μ and σ^2 are the mean and variance of the distribution, respectively.

In this paper, it is assumed that the duration time between any two size changes, denoted $\boldsymbol{T_c}$, is constant, for a part population under continuous disturbance. Also, since size variations are assumed to only occur at part replacement times, the duration time can be equal to multiples of the replacement interarrival time Δt. To facilitate discussion, Δt is set to 1 in the following exemplary simulation.

A random number generator is used to supply $\boldsymbol{n(x;\mu,\sigma)}$ disturbances to the original population size, $\boldsymbol{M}$. Denoting the current population size by $\boldsymbol{M'}$, each random deviation represents a value of $\boldsymbol{100(M'-M)/M}$ at that time, which stays in the range of $\boldsymbol{-3\sigma}$ to $\boldsymbol{3\sigma}$. Thus, when the $|\boldsymbol{3\sigma\%}|$ noise is applied, the size of the non-constant population ranges from $\boldsymbol{M(1-3\sigma\%)}$ to $\boldsymbol{M(1+3\sigma\%)}$. For example, if the random deviation is equal to 2.5, then at that time, the population size is calculated as $\boldsymbol{M(1+2.5/100)=1.025M}$.

In the simulation below, $\mu=0$, $\sigma=1$. The size of the non-constant population is varied for 1,000 time units, with size changes every one time unit, i.e., $\boldsymbol{T_c=1}$, as shown in Figure 5. The replacement rates of this population for two different failure density functions are shown in Figures 6 and 7. As expected, in both cases, the replacement rates reach steady state. These values correspond to the steady state values of the original size populations, if they had been kept as constant-size populations.

A different non-constant-size population, whose size changes every 30 time units, i.e., $\boldsymbol{T_c=30}$, is shown in Figure 8. The replacement rate for this population is examined for the same two failure density functions considered in Figures 6 and 7. The corresponding results are shown in Figures 9 and 10.

When comparing Figure 6 with Figure 9 and Figure 7 with Figure 10, it is observed that the time duration between size variation has no effect on the steady state value of the replacement rate. Since duration between size variation is harder to predict in practice than variation range, this conclusion is important for estimation of replacement rate for a part population under any type of continuous disturbance.

AN EXAMPLE: ANALYSIS OF INDUSTRIAL FAILURE DATA

In the previous section, it was shown that when a continuous disturbance is applied to the population size, the "steady state" replacement rate fluctuates in a limited range around the steady state value of the constant-size population.

In order to illustrate the implications of the above-mentioned phenomenon on practical failure data, some available industrial data were considered in our work.

Actual Failure Data

Failure data collected from one type of commercial systems were available to this research. Each commercial system is identified by its serial number. In each system, there are two parts of the type for which data was collected. The two parts are at different locations in the system, and the locations are represented by socket numbers, 0 and 1. The part population is perfectly maintained. Namely, when a part fails, it is replaced immediately. The failed parts are remanufactured and used as replacements. Each part has a unique serial number that remains unchanged after it is remanufactured.

The commercial system population was tracked for a 24-month period. As shown in Table 1, when a replacement occurs, the following information is recorded: the system serial number (Sys No.), the socket number (Soc No.), the part serial number (Part No.), the date on which the part is installed (Install Date), the date on which the part has failed and is removed (Removal Date), and the total number of days the part has been in operation (Days Last).

Table 1: Practical failure data.

Sys No.	Soc No.	Part No.	Install Date	Removal Date	Days Last
1	1	116	Day 0	Day 4	4
2	0	112	Day 3	Day 14	11
2	1	105	Day 14	Day 34	20
3	0	119	Day 36	Day 49	13
4	0	121	Day 35	Day 53	18
5	0	104	Day 12	Day 53	41
3	0	118	Day 49	Day 56	7
3	1	105	Day 56	Day 64	8
3	0	111	Day 64	Day 79	15
. . .	. . .	. . .	. . .	. . .	. . .
9	0	111	Day 721	Day 733	12

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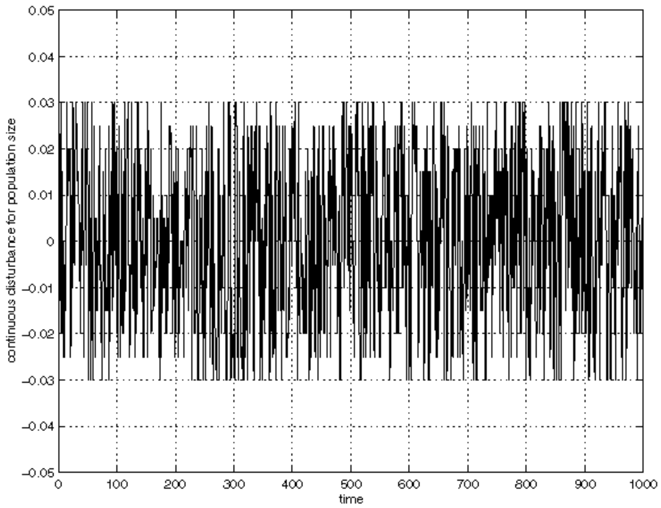

Figure 5: Continuous disturbance ($T_c = 1$).

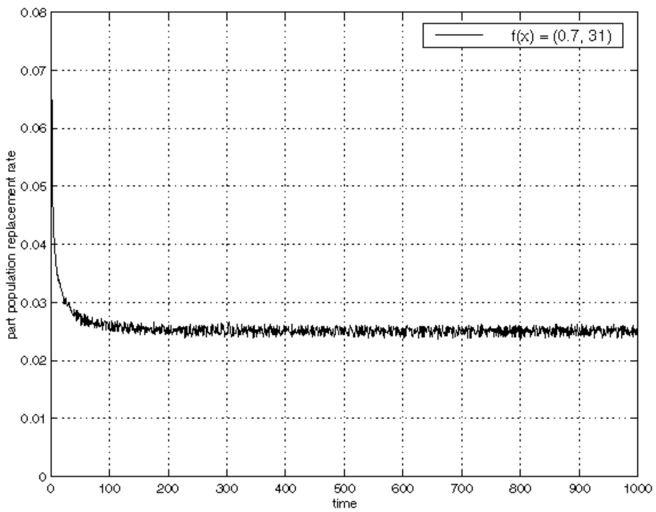

Figure 6: Replacement rate under continuous disturbance ($T_c = 1$, $f(x) = (0.7, 31)$).

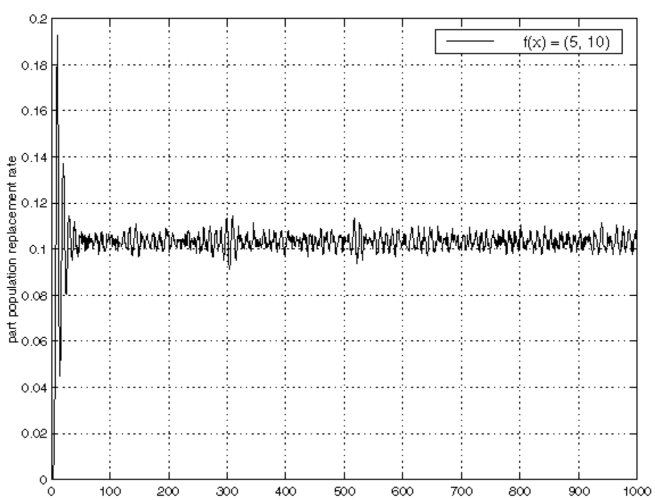

Figure 7: Replacement rate under continuous disturbance ($T_c = 1$, $f(x) = (5, 10)$).

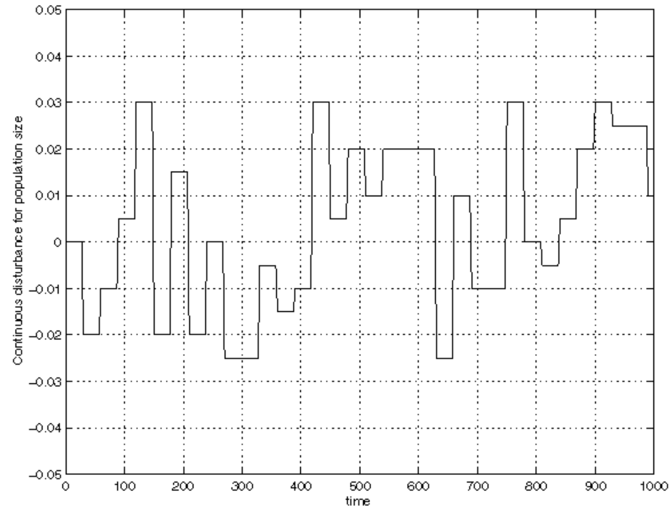

Figure 8: Continuous disturbance ($T_c = 30$).

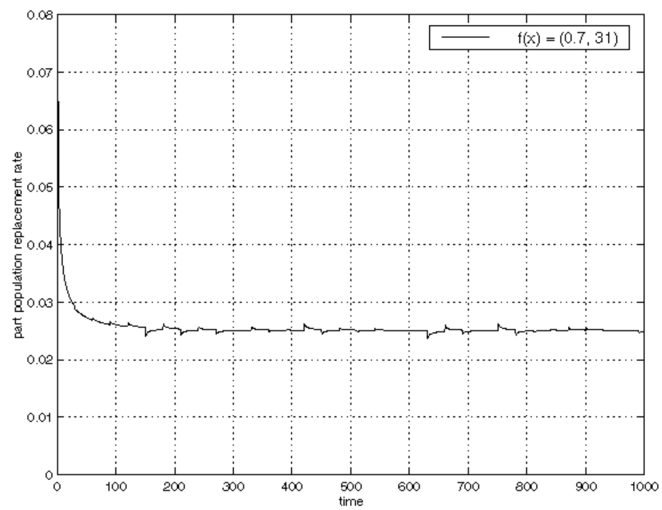

Figure 9: Replacement rate under continuous disturbance ($T_c = 30$, $f(x) = (0.7, 31)$).

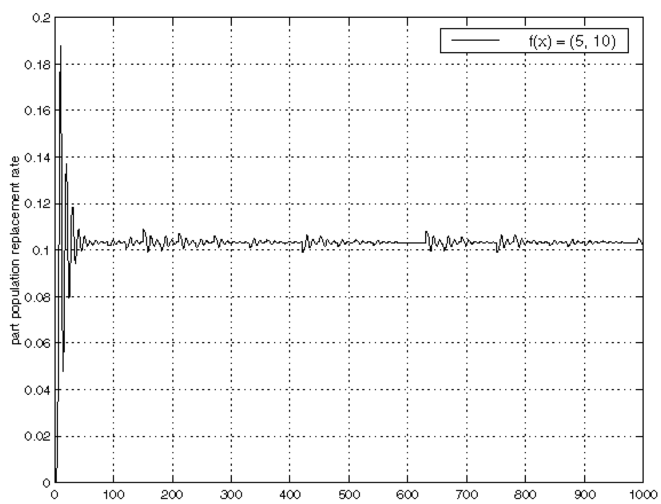

Figure 10: Replacement rate under continuous disturbance ($T_c = 30$, $f(x) = (5, 10)$).

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Replacement Cost Evaluation

The number of failures that occur during $(0,t]$ is denoted $N(t)$. Then, $\{N(t), t \geq 0\}$ is an integer-valued *counting process*, which includes both the number of failures during $(0,t]$ and the instants $T_1, T_2, \ldots,$ at which they occur (Ascher and Feingold, 1984). In this paper, the replacement rate is expressed as:

$$v(t) = \frac{N(t) - N(t - \delta t)}{M \delta t}, \tag{13}$$

where M is the part population size at time t and δt is the interarrival time between two failures.

The replacement rate and costs were calculated for the available failure data as follows. First, the *counting process* $\{N(t), t \geq 0\}$ was obtained, as shown in Figure 11. This was achieved by sorting the data by Removal Date. The day of the first available data was set to $t=0$. The number of failures, i.e., replacements, that occur during $(0,t]$ is counted and denoted $N(t)$. In $\{N(t), t \geq 0\}$, both the number of failures during $(0,t]$ and the instants $T_1, T_2, \ldots,$ at which they occur are recorded.

The cumulative replacement cost from time 0 to t is:

$$C(t) = pN(t), \tag{14}$$

where p is the replacement cost for one part, and $N(t)$ is the number of replacements that occur during $(0,t]$.

Using Equation (3), at time t, the part population replacement cost per unit time is calculated as:

$$c(t) = pMv(t)$$
$$= p\frac{N(t) - N(t - \delta t)}{\delta t}, \tag{15}$$

where δt is the interarrival time between two replacements. For example, let $t=14$ and $N(t)=2$ on Day 14 and let $t=34$ and $N(t)=3$ on Day 34. Thus, from time $t=14$ to time $t=34$, the part population replacement cost per day is calculated as:

$p(3-2)/(34-14) = 0.05p$

Population size is not needed to calculate cumulative replacement cost in Equation (14) nor part population replacement cost per unit time in Equation (15). If $p=1$, the cumulative replacement cost can be obtained using Figure 11.

When calculating the replacement rate using Equation (13), knowledge of the population size is required. The data in Table 1 is therefore resorted according to the system serial number. In our case, the part population size varies daily under continuous disturbance. However, the changes are minor in magnitude when considered in time units of "months". Thus, the part population size was "counted" monthly, or over 30 days.

As previously conjectured, the duration between population size variation has no influence on the steady state value of the replacement rate. Thus, it is anticipated that the population size approximation by months should not affect the estimation of the replacement rate. The actual part population size is plotted in Figure 12, where the time unit is in months.

To calculate the replacement rate for each month, first, the number of the replacements that occurred in that month, denoted V, is determined. Then, the replacement rate is calculated as $V/(30*M)$, where M is the population size in that month. Figure 13 shows the actual replacement rate for each month.

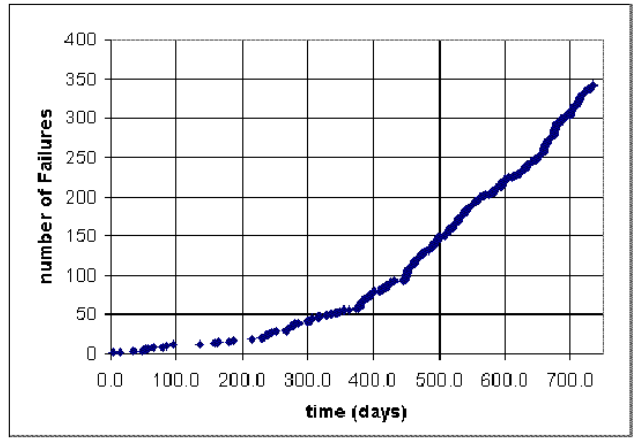

Figure 11: Counting process $\{N(t), t \geq 0\}$.

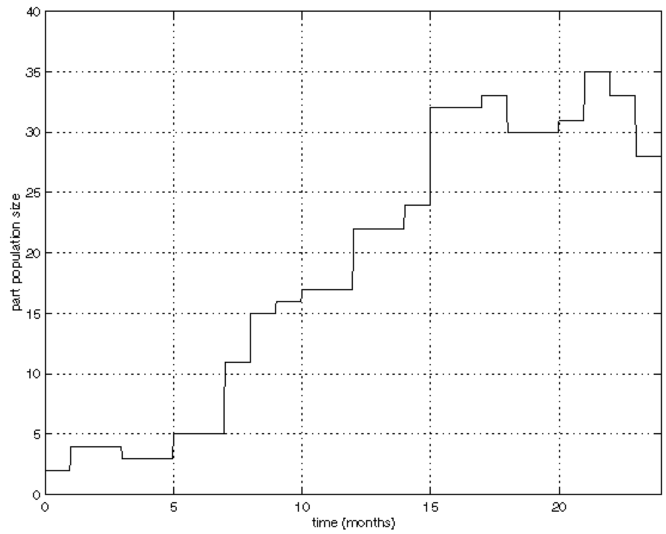

Figure 12: Actual part population size.

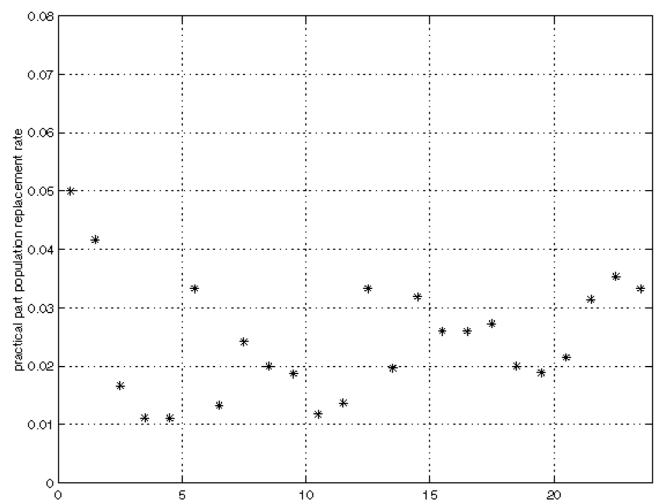

Figure 13: Actual part population replacement rate.

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Simulating the Replacement Process

To generate a simulated replacement process that corresponds to the actual process, the failure density function of the actual parts involved was obtained by curve fitting. Subsequently, this failure density function was used to generate the simulated replacement process under continuous disturbance. Since ***80%*** of the replacement parts over the time period tracked were the same type, a single Weibull failure probability function, Equation (6), was used to fit all the failure data. The curve-fitting result is shown in Figure 14, where $\alpha=0.7044$ and $\beta=31.2725$ with a Coefficient of Multiple Determination of $R^2=0.998$. For the example presented, there was no clear system modification process that involved replacement of one type of parts with a different type of parts. Therefore, the capability of simulating this behavior was not used to generate the replacement rate that approximates the actual replacement process. The reliability model for the replacement process without system modifications, Equation (1), where $f_1(x)=f_2(x)=f(x)$, is applicable here.

A simulation of the replacement process, under continuous disturbance, was run to generate the replacement rate for the part population. Since the failed parts are replaced immediately, the part population is perfectly maintained. Thus, the scheduled replacement interarrival $\Delta t \rightarrow 0$. Herein, Δt is set to 0.1.

Figure 15, showing the plots of both the simulated and actual replacement rates, illustrates that the actual replacement rate fluctuates within a range similar to that for the simulation results. Thus, it can be concluded that the replacement rate of an industrial process can be approximated using the inference models presented in this paper with a limited set of actual failure data.

SUMMARY

In this paper, the replacement rate behavior of a non-constant-size population of parts was studied to facilitate estimation of life-cycle replacement cost in the design stage. The size change is classified as a pulse disturbance or a continuous disturbance. A reliability model developed in previous work was modified to accommodate population size changes. For a pulse disturbance, it is shown that the replacement rate experiences a transient behavior but eventually reaches steady state. For a continuous disturbance, it is shown that the steady state value of replacement rate varies but is centered at the steady state value for the corresponding constant-size population. Furthermore, the time duration between size variation has little influence on the centerline of the replacement rate.

Actual failure data, collected from a part population replacement process under continuous disturbance, were analyzed. Using a counting process, the replacement rate was calculated. The failure density function for the part population was obtained through curve fitting. This example showed that the reliability model for a part population replacement process under continuous disturbance could be used to approximate an actual replacement process.

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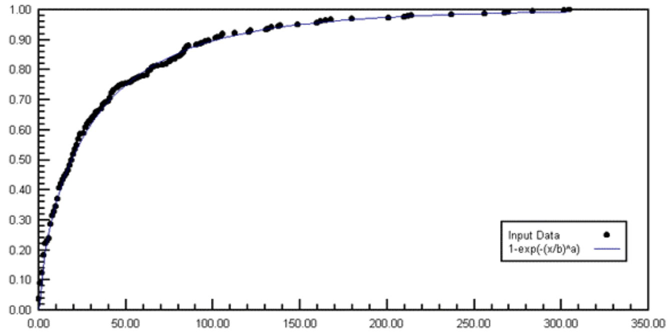

Figure 14: Curve fitting of Weibull failure probability function.

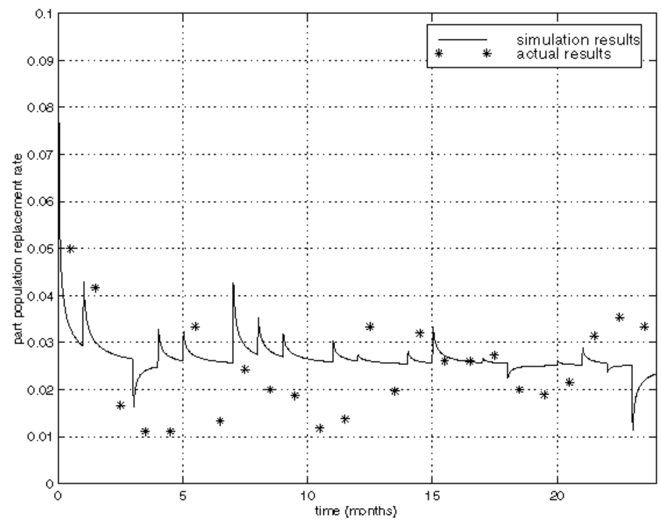

Figure 15: Comparing actual replacement rate with simulation results.

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