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Reliability Analysis of Non-Constant-Size Part Populations in Design for Remanufacture

Remanufacture offers significant economic and ecological advantages over other end-of-life options for appropriate products. The goal of this research is to estimate replacement requirements of parts in systems that are remanufactured. In our previous work, a novel repairable-system reliability model that allows system modifications was developed to describe a population of systems that are remanufactured. In this paper, the reliability model is modified to accommodate changes to the population size, while the population is in service, to better describe actual processes. The effects of two types of disturbances to population size, pulse and continuous, on the replacement rate behavior are studied. Analysis of actual industrial data is presented as an example of population replacement under continuous disturbance. This example confirms that a simulation using the reliability model described in this paper yields an estimate for replacement rate with acceptable error bound. [S1050-0472(00)00302-0]

Introduction

This research is in the area of environmentally conscious product design and manufacture. An overall approach to environmental evaluation of products is Life-Cycle Analysis (LCA), which tracks resource inputs and outputs required for a product from material extraction to end-of-life disposition [1]. Options to disposal at product end-of-life include scrap-material recycling, remanufacture and repair/reuse.

Remanufacture offers significant economic and ecological advantages over other end-of-life options for appropriate products. Remanufacture consists of the production-batch disassembly, cleaning, replacement and refurbishment of worn parts in defective or obsolete products. It restores worn and discarded durable products to a like-new condition [2]. Remanufacture can be considered as recycling at the component level instead of the material level. Therefore, it avoids the possibly unnecessary resource consumption of scrap-material recycling while preserving the value added to a component during manufacture [3]. The production-batch and off-site nature of remanufacture results in a labor cost significantly lower than that required for individual repair. Remanufacture is estimated to save between 40 and 60 percent of the cost of manufacturing a completely new product, while requiring only 20 percent of the energy [4].

Research that addresses design-for-reuse includes work on enhancing reliability through design [5]. Specifically related to remanufacturing, Hammond and Bras [6] generated design-for-remanufacture guidelines and metrics.

Since the primary goal in remanufacture is the reuse of parts, and reliability is critical to determining reusability of parts, a reliability model that more accurately describes remanufacture was developed in previous work [7,8]. This reliability model assumed a constant population size. In practice, however, population sizes may vary as a function of time. Some systems may not be replaced after failure, causing a decrease in population size, while other systems may be added after the initial population was put into service, causing an increase in population size. Such system populations are referred to herein as non-constant-size system populations.

A system is comprised of parts. Thus, a population of systems is modeled as a collection of populations of parts. In this paper, the replacement rate behavior of a non-constant-size part population is studied.

Reliability Model for Systems Undergoing Remanufacture

This section summarizes the reliability model that was developed in previous work [7,8].

This reliability model for systems undergoing remanufacture describes a population of m-socket series systems. The population size is constant. The population of M systems is represented as a collection of m populations of constituent parts, and parts are treated as members of their respective populations.

In remanufacture, part failure results in replacement of the part with a component of the same or different type. The remaining system either remains unchanged or is reconfigured to accommodate the replacement part. The failure density function of the original parts in the population is denoted $f_1(x)$. The failure density function of the replacement parts is denoted $f_2(x)$. Thus, the replacement process of a constant-size part population is denoted $\{f_1(x)\leftarrow f_2(x)\}$.

If $f_1(x)$ and $f_2(x)$ are the same, i.e., $f_1(x)=f_2(x)=f(x)$, then system modification does not occur through the replacement process. However, if $f_2(x)$ and $f_1(x)$ differ, then system modification does occur during the replacement process.

The age distributions of each of the part populations are tracked to determine the reliability of the composite system population. The fractions of parts in the population, whose ages equal $i\Delta t$, where $i=0,1,\ldots n$, at time $t_n=n\Delta t$, are:

$$q_n(t_n)=q_0(t_0)\left[1-\int_0^{n\Delta t} f_1(x)dx\right]$$

$$q_{n-1}(t_n)=q_0(t_1)\left[1-\int_0^{(n-1)\Delta t} f_2(x)dx\right]$$

$$\vdots$$

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$$q_2(t_n)=q_0(t_{n-2})\left[1-\int_0^{2\Delta t} f_2(x)dx\right]$$

$$q_1(t_n)=q_0(t_{n-1})\left[1-\int_0^{\Delta t} f_2(x)dx\right]$$

$$q_0(t_n)=q_0(t_0)\left[\int_{(n-1)\Delta t}^{n\Delta t} f_1(x)dx\right] + q_0(t_1)\left[\int_{(n-2)\Delta t}^{(n-1)\Delta t} f_2(x)dx\right]+\ldots + q_0(t_{n-2})\left[\int_{\Delta t}^{2\Delta t} f_2(x)dx\right] + q_0(t_{n-1})\left[\int_0^{\Delta t} f_2(x)dx\right]. \tag{1}$$

After obtaining the value of $q_0(t_n)$, the fraction of parts failed and replaced at time t_n, the replacement rate at time $t_n=n\Delta t$, i.e., $v(n\Delta t)$, is calculated as:

$$v(t_n)=\frac{q_0(t_n)}{\Delta t} \tag{2}$$

Jiang et al. [8] investigated the stochastic behavior of the reliability model, and theoretically proved that the replacement process with system modifications under a perfectly maintained repair policy reaches steady state, and is a Superimposed Renewal Process (SRP). At the limit, as $t\rightarrow\infty$, a SRP behaves like a Homogeneous Poisson Process (HPP) with a constant hazard rate, which can be represented by an exponential distribution [9]. In a perfectly maintained process with system modifications, the steady-state value for replacement rate depends only on the failure density function of the replacement parts. Specifically, it converges to the reciprocal of the mean life of the replacement parts.

Representation of Failure Density Function. In the reliability model represented in Eq. (1), there exists no restriction on the failure density function $f(x)$. In this paper, the two-parameter Weibull distribution is used to represent the failure density function $f(x)$ for simulation purposes. Extension to other distributions is fairly straightforward.

The two-parameter Weibull distribution is:

$$f(x)=\begin{cases}\dfrac{\alpha x^{\alpha-1}}{\beta^\alpha}\exp\left[-\left(\dfrac{x}{\beta}\right)^\alpha\right], & 0\leqslant x\leqslant\infty\\ 0, & \text{elsewhere,}\end{cases} \tag{3}$$

where $\alpha>0$, $\beta>0$, and x is the age of the part.

The Weibull failure probability function is:

$$i(x)=\int_0^x f(x)dx=-\exp\left(-\left(\frac{x}{\beta}\right)^\alpha\right)\Bigg|_0^x=1-\exp\left(-\left(\frac{x}{\beta}\right)^\alpha\right). \tag{4}$$

Reliability Model for Non-Constant-Size Populations

In this section, the reliability model is modified to accommodate population-size changes during the replacement process. The variation of a population's size can be treated as a random disturbance. Two types of disturbances are considered:

1 Pulse disturbance, where the variation occurs only once during the time under consideration, and
2 Continuous disturbance, where population-size variations occur at random time intervals and at random amplitudes (e.g., Fig. 7).

Population-size Increases. For a replacement process where the population size increases, the total population, (i.e., the part population after the size increase), can be viewed as a combination of two separate populations. The first is the original part population, installed at time t_0, whose fraction in the total population after the size increase is r_1. The second population, added at time t_m, has fraction in the total population r_2, where $r_1+r_2=1$.

The replacement process for the original part population starts at time t_0, while the replacement process for the newly added part population starts at time t_m. After new parts have been added, the replacement rate behavior for the total population is a combination of the two replacement processes.

Let us suppose that a time period of $n\Delta t$ has passed after the new parts have been added. The replacement rate at time $t_{m+n}=(m+n)\Delta t$ is to be calculated. In the total population, the fraction of parts which failed during time $t_{m+n}=(m+n)\Delta t$ to $t_{m+n+1}=(m+n+1)\Delta t$ can be expressed as:

$$q_0(t_{m+n})=r_1q_0^1(t_{m+n})+r_2q_0^2(t_n) \tag{5}$$

where $q_0^1(t_{m+n})$ is the fraction of parts that failed of the original part population and $q_0^2(t_n)$ is the fraction of parts that failed of the newly added part population. $q_0^1(t_{m+n})$ and $q_0^2(t_n)$ are calculated using Eq. (1). According to Eq. (2), at time t_{m+n}, the replacement rate for the entire population is

$$v((m+n)\Delta t)=\frac{q_0(t_{m+n})}{\Delta t}=\frac{r_1q_0^1(t_{m+n})+r_2q_0^2(t_n)}{\Delta t} \tag{6}$$

Population-size Decreases. During the part population replacement process, the population size may also decrease due to the removal of a fraction of parts for various reasons. For example, some systems may be discontinued and removed. If the replacement rate for above case is considered, the age groups to which the removed parts belong must be specified. Otherwise, the information for the part population age distribution cannot be determined.

Herein, only one condition for size decrease is considered, i.e., some of the parts that failed are not replaced. Those unreplaced failed parts are removed from the original population. Thus, the population size is decreased by a fraction that cannot be greater than the fraction of parts which failed at that time.

For a part population whose size decreases, the original part population can be viewed as a combination of two separate part populations. The first one is the population of parts remaining in the process after the size decrease, whose fraction in the original part population is s_1. The second one is the part population that failed at time t_m, and is not replaced, whose fraction in the original population is s_2, where $s_1+s_2=1$. Note that, s_2 cannot be greater than the fraction of parts failed at t_m.

For the sake of discussion, let us suppose that the removed parts were actually replaced, namely, added back into the process as a new population. Then, they would be experiencing a replacement process that starts at time t_m, whereas the replacement process for the original part population had started at time t_0. Similar to the replacement process when the population size increases, the remaining part population and the removed (but instantly re-added) part population will experience different replacement processes.

Suppose a period of $n\Delta t$ has passed after the new parts are "pseudo-added," and the replacement rate at time $t_{m+n}=(m+n)\Delta t$ is to be calculated. For the remaining population, the fraction of parts that failed during time $t_{m+n}=(m+n)\Delta t$ to $t_{m+n+1}=(m+n+1)\Delta t$ can be expressed as

$$q_0(t_{m+n})=\frac{(q_0^1(t_{m+n})-s_2q_0^2(t_n))}{s_1}, \tag{7}$$

where $q_0^1(t_{m+n})$ is the fraction of parts that failed in the original part population and $q_0^2(t_n)$ is the fraction of parts that failed in the pseudo-added part population. Then, at time t_{m+n}, the replacement rate of the remaining population is

$$v((m+n)\Delta t)=\frac{q_0(t_{m+n})}{\Delta t}=\frac{(q_0^1(t_{m+n})-s_2q_0^2(t_n))}{s_1\Delta t}, \qquad (8)$$

where $q_0^1(t_{m+n})$, $q_0^2(t_n)$ are calculated using Eq. (1).

The use of the pseudo-added failure group simplifies the calculation of the fraction of the population that fails at a given time, which is used to determine the failure rate. To calculate the numerator terms of Eq. (7), the last term of Eq. (1) is used to compute $q_0(t_i)$. Equation (1) shows that the calculation of each $q_0(t_i)$ is computationally expensive and requires values for $q_0(t_0)$ to $q_0(t_{i-1})$, previously determined. By using the pseudo-added concept, we can reuse the values for each $q_0(t_i)$ calculated before the population-size decrease, even after a part of the population has been removed.

Alternatively, without the use of the pseudo-added concept, the removal of the failure group could be modeled directly. After each population-size decrease, to calculate the fraction of the population that fails at each time step, the fractions of the age groups remaining in service must be adjusted. For continuous disturbance, these adjustments must be made after each population-size change.

The concept of the pseudo-added population also allows the direct use of the constant-size model for a population with size decreases, consistent with the treatment of a population with size increases.

Reliability Analysis for Pulse Disturbance

In this section, only a replacement process under pulse disturbance causing size increase will be discussed. The analogy for pulse disturbance causing size decrease is straightforward and detailed in [10].

Analysis of Replacement Process. When a new sub-population of systems is added to a population, the configuration of the new systems may be the same as, or different from, that of the original population. Thus, for a part population under consideration, the failure density function of those newly added parts may be the same as that of the initial part population, or different. Assuming that when a different type of parts replaces failed parts, the original type of parts will not be reintroduced into the population, three situations may exist:

1 The process begins with $\{f(x)\leftarrow f(x)\}$. At time t_m, $m>0$, new parts with a failure density function $f(x)$ are added into the original part population. After time t_m, failed parts continue to be replaced with new parts with the same failure density function, $f(x)$. This process, denoted $\{\{f(x)\leftarrow f(x)\},+f(x), f(x)\}$, consists of three components, the original replacement process, $\{f(x)\leftarrow f(x)\}$, the addition of new parts, $+f(x)$, and the failure density function of the replacement parts after size increase, $f(x)$.

2 The process begins with $\{f_1(x)\leftarrow f_1(x)\}$. At time t_m, $m>0$, new parts with a failure density function $f_1(x)$ are added into the original part population. After time t_m, failed parts are replaced with new parts with a different density function, $f_2(x)$. This process is denoted $\{\{f_1(x)\leftarrow f_1(x)\},+f_1(x),f_2(x)\}$. Note that in this process, the replacement policy changes from replacement-with-same to replacement-with-different type of parts, after the new sub-population of parts is added.

3 The process begins with $\{f_1(x)\leftarrow f_2(x)\}$. At time t_m, $m>0$, new parts with a failure density function $f_2(x)$ are added into the original part population. After time t_m, failed parts are replaced with new parts with a failure density function $f_2(x)$. This process is denoted $\{\{f_1(x)\leftarrow f_2(x)\},+f_2(x),f_2(x)\}$.

When the replacement policy changes after the new parts are added, as in Case 2, the reliability model must be modified as follows:

$$q_i(t_{m+n})=q_0(t_{n+m-i})\left[1-\int_0^{i\Delta t}f_1(x)dx\right],$$

$$i=n+1,\ldots,n+m$$

$$q_i(t_{m+n})=q_0(t_{n+m-i})\left[1-\int_0^{i\Delta t}f_2(x)dx\right],$$

$$i=1,\ldots,n$$

$$q_0(t_{m+n})=\sum_{j=n+1}^{n+m}q_0(t_{n+m-j})\int_{(j-1)\Delta t}^{j\Delta t}f_1(x)dx+\sum_{j=1}^{n}q_0(t_{n+m-j})\int_{(j-1)\Delta t}^{j\Delta t}f_2(x)dx \qquad (9)$$

As stated in [8], for a perfectly maintained process with $\{f_1(x)\leftarrow f_2(x)\}$, the replacement rate will reach a constant value that equals the reciprocal of the mean life of the replacement parts. Thus, it can be conjectured that the replacement rate of a process subject to pulse disturbance will also reach a steady-state value that depends only on the replacement parts.

The age distribution of a part population at steady state can be calculated using expressions developed in [8]. When new parts are added into this original part population via a pulse disturbance, the age distribution of the population is subjected to a transient period until the new part population reaches steady state.

Examples. Simulations were run to track the replacement rate behavior for a non-constant-size part population with a pulse size increase. To show the effect of a size change more clearly, time t_m was chosen as a time instance after the original population has reached steady state.

Replacement rates for the above three processes, $\{\{f(x)\leftarrow f(x)\},f(x),f(x)\}$, $\{\{f_1(x)\leftarrow f_1(x)\},+f_1(x),f_2(x)\}$, and $\{\{f_1(x)\leftarrow f_2(x)\},+f_2(x),f_2(x)\}$ are shown in Figs. 1, 2 and 3, respectively, for three different replacement ratios, r_1. As previously stated, r_1 is the fraction of original population with respect to the overall population size after the addition of new parts. In Fig. 1, the failure density function $f(x)$ is (3,4), i.e., α=3, β=4. In Fig. 2, $f_1(x)$ is (3,4) and $f_2(x)$ is (3,10). In Fig. 3, $f_1(x)$ is (10,10) and $f_2(x)$ is (3,10). Time t_m is set to 40, 60 and 60 in Figs. 1, 2, and 3, respectively.

As expected, in Figs. 1 through 3, the replacement rate exhibits transient behavior when new parts are added. Since the original

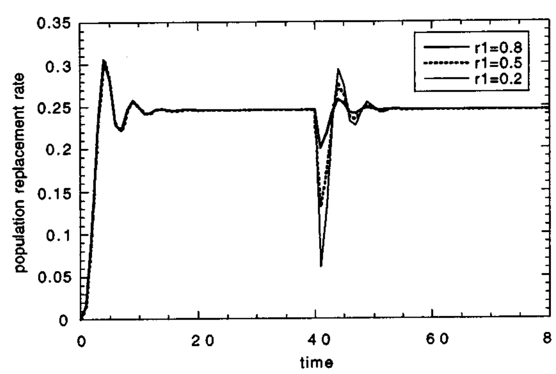

Fig. 1 Replacement rates for $\{\{f(x)\leftarrow f(x)\},+f(x),f(x)\}$, $f(x)=(3,4)$

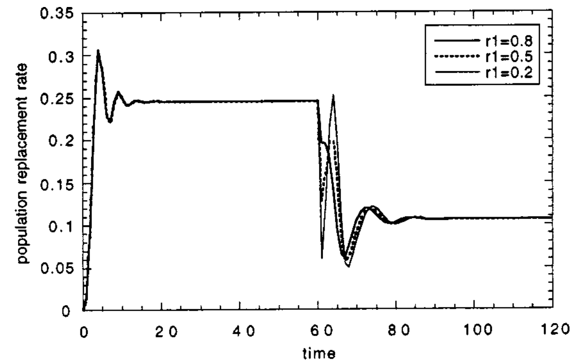

Fig. 2 Replacement rates for $\{\{f_1(x)\leftarrow f_1(x)\},+f_1(x),f_2(x)\}$, $f_1(x)=(3,4)$, $f_2(x)=(3,10)$

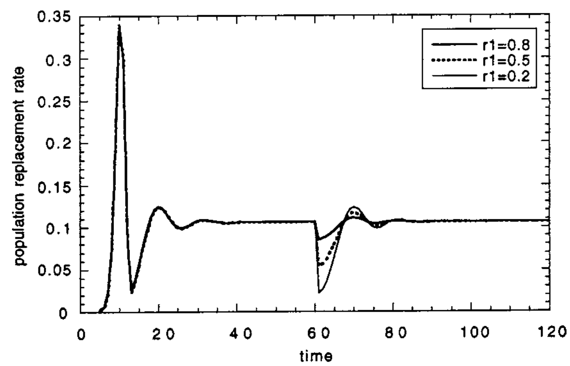

Fig. 3 Replacement rates for $\{\{f_1(x)\leftarrow f_2(x)\},+f_2(x),f_2(x)\}$, $f_1(x)=(10,10)$, $f_2(x)=(3,10)$

part population has already reached steady state, it can be concluded that the transient behavior is caused solely by the addition of the parts.

The initial high oscillations in replacement rate, before the new population is added, have long been known to be due to a small standard deviation in the failure density function. For the Weibull distribution, the standard deviation in time to failure decreases with increasing values of α. These oscillations are damped out over time as the population becomes more age-diversified [7]. With the added population, the ratio of the original population to the total population, after the new parts are added, also affects the degree of the disturbance due to the population-size increase. In Fig. 1, all parts have the same failure density function. Therefore, a larger r_1 corresponds to more original parts that have already reached steady state to absorb the disturbance of the transient state of the added population. In Fig. 2, the steady-state replacement rate (inversely proportional to β) of the parts used for replacement after the population-size increase, differs from that of the parts before the size increase. In Fig. 3, the degree of oscillation in the transient state, proportional to α, of the replacement parts is lower than for the original parts.

Reliability Analysis for Continuous Disturbance

In this section, the replacement rate behavior of a part population under continuous disturbance is studied.

The previous section showed that when a pulse disturbance is applied to a replacement process, the replacement rate exhibits a transient behavior. It can be conjectured that the replacement rate under continuous disturbance will also fluctuate since a continuous disturbance can be viewed as a sequence of pulse disturbances.

Example. Let the original part population size be denoted by M. Let the deviations from the population size M be normally distributed,

$$n(x;\mu,\sigma)=\frac{1}{\sqrt{2\pi}\sigma}e^{-(1/2)[(x-\mu)/\sigma]^2},\quad -\infty<x<\infty, \tag{10}$$

where μ and σ^2 are the mean and variance of the distribution, respectively.

In this paper, it is assumed that the duration time between any two size changes, denoted T_c, is constant, for a part population under continuous disturbance. Also, since size variations are assumed to only occur at part replacement times, the duration time is equal to multiples of the replacement interarrival time Δt. To facilitate discussion, Δt is set to 1 in the following simulation.

A random number generator is used to supply $n(x;\mu,\sigma)$ disturbances to the original population size, M. Denoting the current population size M', each random deviation represents a value of $100(M'-M)/M$ at that time, which stays in the range of -3σ to 3σ. Thus, when the $|\pm 3\sigma\%|$ noise is applied, the size of the non-constant population ranges from $M(1-3\sigma\%)$ to $M(1+3\sigma\%)$. For example, if the random deviation is equal to 2.5, then at that time, the population size is calculated as $M(1+2.5/100)=1.025M$.

In the simulation below, $\mu=0$, $\sigma=1$. The size of the non-constant population is varied for 1,000 time units, with size changes every one time unit, i.e., $T_c=1$, as shown in Fig. 4. The replacement rates of this population for two different failure density functions are shown in Figs. 5 and 6. As expected, in both cases, the replacement rates reach steady state. These values correspond to the steady-state values of the original size populations, if they had been kept as constant-size populations.

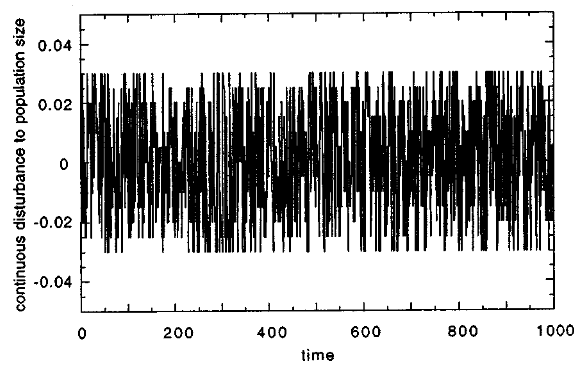

Fig. 4 Continuous disturbance ($T_c=1$)

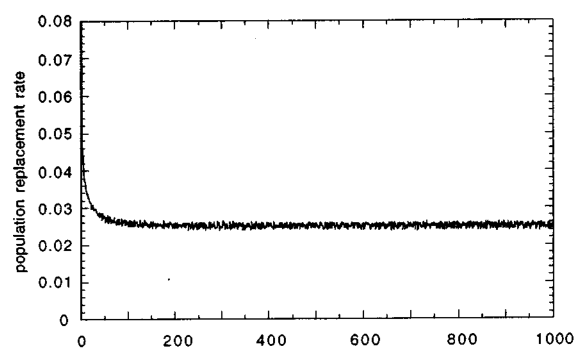

Fig. 5 Replacement rate under continuous disturbance ($T_c=1$, $f(x)=(0.7,31)$)

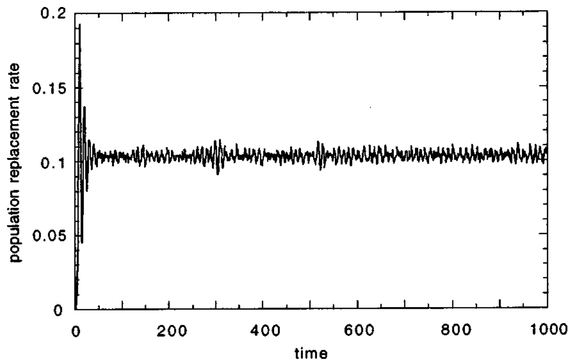

Fig. 6 Replacement rate under continuous disturbance (T_c=1, $f(x)$=(5,10))

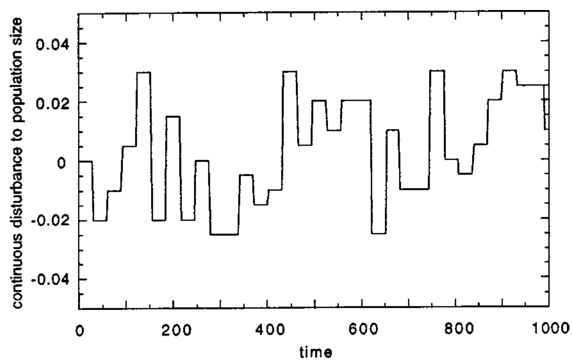

Fig. 7 Continuous disturbance (T_c=30)

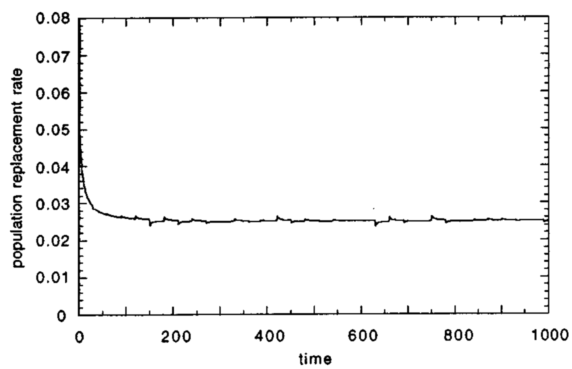

Fig. 8 Replacement rate under continuous disturbance (T_c=30, $f(x)$=(0.7,31))

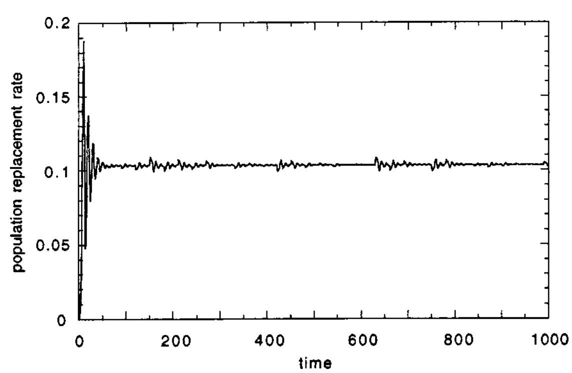

Fig. 9 Replacement rate under continuous disturbance (T_c=30, $f(x)$=(5,10))

A different non-constant-size population, whose size changes every 30 time units, i.e., $T_c = 30$, is shown in Fig. 7. The replacement rate for this population is examined for the same two failure density functions considered in Figs. 5 and 6. The corresponding results are shown in Figs. 8 and 9.

When comparing Fig. 5 with Fig. 8 and Fig. 6 with Fig. 9, it is observed that the time duration between size variation has no effect on the steady-state value of the replacement rate. Since duration between size variation is harder to predict in practice than variation range, this conclusion is important for estimation of replacement rate for a part population under any type of continuous disturbance.

An Example: Analysis of Industrial Failure Data

In the previous section, it was shown that when a continuous disturbance is applied to the population size, the ''steady-state'' replacement rate fluctuates in a limited range around the steady-state value of the constant-size population.

To illustrate the implications of the above-mentioned phenomenon on practical failure data, some available industrial data were considered in our work.

Description of Failure Data. Failure data collected from one type of commercial systems were available to this research. Each commercial system is identified by its serial number. In each system, there are two parts of the type for which data was collected. The two parts are at different locations in the system, and the locations are represented by socket numbers, 0 and 1. The part population is perfectly maintained. Namely, when a part fails, it is replaced immediately. The failed parts are remanufactured and used as replacements. Each part has a unique serial number that remains unchanged after it is remanufactured.

The commercial system population was tracked for a 24-month period. As shown in Table 1, when a replacement occurs, the following information is recorded: the system serial number (Sys No.), the socket number (Soc No.), the part serial number (Part No.), the date on which the part is installed (Install Date), the data on which the part has failed and is removed (Removal Date), and the total number of days the part has been in operation (Days Last).

Replacement Rate Evaluation. The number of failures that occur during $(0,t]$ is denoted $N(t)$. Then, $\{N(t), t \geqslant 0\}$ is an integer-valued *counting process*, which includes both the number of failures during $(0,t]$ and the instants $T_1, T_2, \ldots,$ at which they occur [11]. In this paper, the replacement rate is expressed as:

$$v(t) = \frac{N(t) - N(t - \delta t)}{M\,\delta t}, \qquad (11)$$

where M is the part population size at time t and δt is the inter-arrival time between two failures.

Table 1 Practical failure data

Sys No.	Soc No.	Part No.	Install Date	Removal Date	Days Last
1	1	116	Day 0	Day 4	4
2	0	112	Day 3	Day 14	11
2	1	105	Day 14	Day 34	20
3	0	119	Day 36	Day 49	13
4	0	121	Day 35	Day 53	18
5	0	104	Day 12	Day 53	41
3	0	118	Day 49	Day 56	7
3	1	105	Day 56	Day 64	8
3	0	111	Day 64	Day 79	15
⋮	⋮	⋮	⋮	⋮	⋮
9	0	111	Day 721	Day 733	12

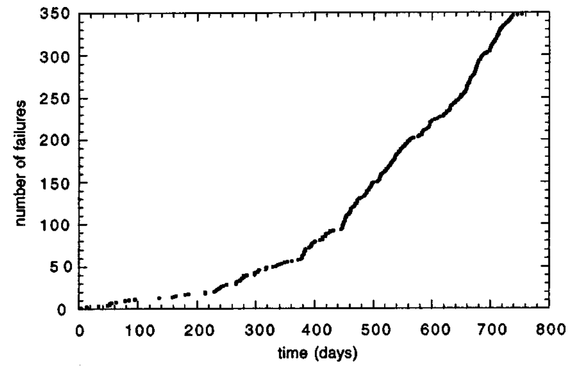

Fig. 10 Counting process $\{N(t), t \geq 0\}$

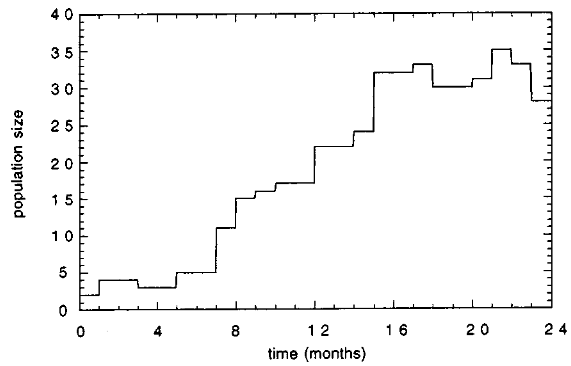

Fig. 11 Population size of data

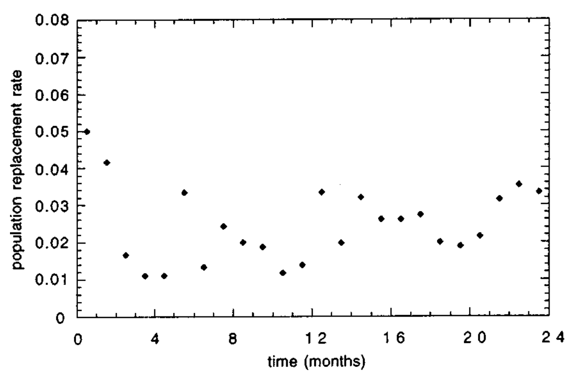

Fig. 12 Population replacement rate of data

The replacement rate was calculated for the available failure data as follows. First, the counting process $\{N(t), t \geq 0\}$ was obtained, as shown in Fig. 10. This was achieved by sorting the data by Removal Date. The day of the first available data was set to $t=0$. The number of failures, i.e., replacements, that occur during $(0,t]$ is counted and denoted $N(t)$. In $\{N(t), t \geq 0\}$, both the number of failures during $(0,t]$ and the instants $T_1, T_2, \ldots,$ at which they occur are recorded.

When calculating the replacement rate using Eq. (11), knowledge of the population size is required. The data in Table 1 is therefore resorted according to the system serial number. In our case, the part population size varies daily under continuous disturbance. However, the changes are minor in magnitude when considered in time units of months. Thus, the part population size was counted monthly, or over 30 days.

As previously conjectured, the duration between population-size variation has no influence on the steady-state value of the replacement rate. Thus, the population-size approximation by months should not affect the estimation of the replacement rate. The actual part population size is plotted in Fig. 11, where the time unit is in months.

To calculate the replacement rate for each month, first, the number of the replacements that occurred in that month, denoted V, is determined. Then, the replacement rate is calculated as $V/(30\,M)$, where M is the population size in that month. Figure 12 shows the actual replacement rate for each month.

Replacement Process Simulation. To generate a simulated replacement process that corresponds to the actual process, the failure density function of the actual parts involved was obtained by curve fitting. Subsequently, this failure density function was used to generate the simulated replacement process under continuous disturbance.

Since 80 percent of the replacement parts over the time period tracked were the same type, a single Weibull failure probability function, Eq. (4), was used to fit all the failure data. The curve-fitting result is shown in Fig. 13, where $\alpha=0.7044$ and $\beta=31.2725$ with a Coefficient of Multiple Determination of $R^2=0.998$. For the example presented, there was no clear system modification process that involved replacement of one type of parts with a different type of parts. Therefore, the capability of simulating this behavior was not used to generate the replacement rate that approximates the actual replacement process. The reliability model for the replacement process without system modifications, Eq. (1), where $f_1(x)=f_2(x)=f(x)$, is applicable here.

A simulation of the replacement process, under continuous disturbance, was run to generate the replacement rate for the part population. Since the failed parts are replaced immediately, the part population is perfectly maintained. Thus, the scheduled replacement interarrival $\Delta t \rightarrow 0$. Herein, Δt was set to 0.1.

Figure 14, showing the plots of both the simulated and actual

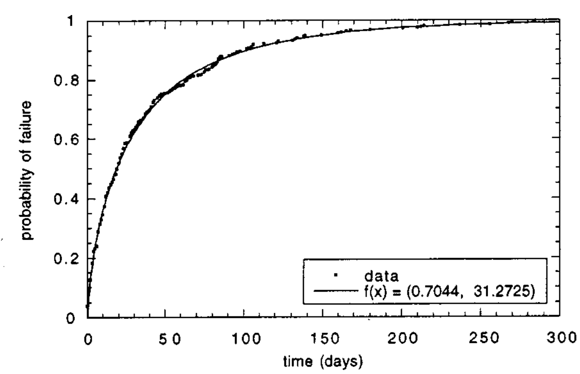

Fig. 13 Curve fitting of Weibull failure probability function

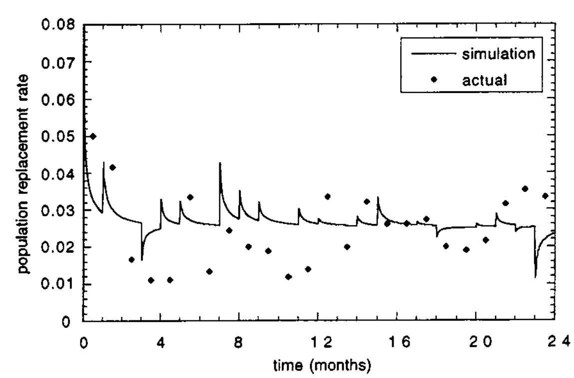

Fig. 14 Replacement rate: data vs. simulation

replacement rates, illustrates that the actual replacement rate fluctuates within a range similar to that for the simulation results. Thus, it can be concluded that the replacement rate of an industrial process can be approximated using the inference models presented in this paper with a limited set of actual failure data.

Summary

In this paper, the replacement rate behavior of a non-constant-size population of parts was studied to facilitate estimation of life-cycle replacement requirements for a population of systems that are remanufactured. A reliability model developed in previous work was modified to accommodate population-size changes. Size changes to a population of parts are classified as pulse disturbances or continuous disturbances. For pulse disturbances, it was shown that the replacement rate experiences a transient behavior but eventually reaches steady state. For continuous disturbances, it was shown that the steady-state value of replacement rate varies but is centered at the steady-state value for the corresponding constant-size population. Furthermore, the time duration between size variation has little influence on the centerline of the replacement rate.

Actual failure data, collected from a part population replacement process under continuous disturbance, were analyzed. Using a counting process, the replacement rate was calculated. The failure density function for the part population was obtained through curve fitting. This failure density function was then used to simulate the corresponding replacement process to compare to the failure data. This example showed that the reliability model for a part population replacement process under continuous disturbance could be used to approximate an actual replacement process.

Acknowledgments

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Nomenclature

The terms used in the reliability modeling and statistical analysis of repairable systems are:

Terms defined for parts

x = age of part, measured in time, a real variable

f(x) = failure density function of parts

$$\int_0^\infty f(x)dx=1$$

i(x) = failure probability function of parts

$$i(x)=\int_0^x f(x)dx$$

$\mathbf{f_1(x)}$ = failure density function of original parts in population

$\mathbf{f_2(x)}$ = failure density function of replacement parts

$\boldsymbol{\alpha}$ = shape parameter of Weibull distribution

$\boldsymbol{\beta}$ = scale parameter of Weibull distribution

Terms defined for part populations

m = number of sockets in a series system

M = number of parts in part population

t = time elapsed from when population of parts was first put into service

Δt = interarrival time between scheduled replacements

v(t) = at time t, replacement rate of part population, i.e., average fraction of parts in population being replaced per unit time

$\mathbf{t_n}$ = denotation for time measured in units of Δt, where n is a non-negative integer defined as $n = t_n/\Delta t$

$\mathbf{q_i(t_n)}$ = age distributions, i.e., at time t_n, fraction of parts with age $i\Delta t$, $i=0,1,\ldots n$

N(t) = number of failures that occur during $(0,t]$

$\mathbf{\{f_1(x)\leftarrow f_2(x)\}}$ = replacement process of a constant-size part population, where original parts have a failure density function $f_1(x)$ and replacement parts have a failure density function $f_2(x)$

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